FERROTOROIDIC DOMAIN PAIRS FOR 116 FERROIC SPECIES

T SREENIVAS*, S UMADEVI**, M VIJAYA LAXMI***
*(Department of Engineering mathematics, A.U.College of Engineering, Andhra University, A.P, India,
** (Department of Engineering mathematics, A.U.College of Engineering, Andhra University, A.P, India,
*** (Department of Engineering mathematics, A.U.College of Engineering, Andhra University, A.P, India)

Abstract:
A ferroic crystal contains one or more domains, volumes of the same homogeneous crystalline structure in different spatial orientations. The homogeneous bulk structure of domains are called domain states. In addition to the ferroelectric, ferromagnetic, and ferroelastic primary ferroic crystals, where domain states differ, respectively by spontaneous polarization, magnetization, and strain, recently observed fourth type of primary ferroic crystal. where domain states differ in spontaneous toroidal moment. This leads to additional contributions to the difference in the free enthalpy between domain states. In this paper, Ferrotoroidic domain pairs are calculated by using coset decomposition for **116 ferroic species**, taking 32 point groups as prototypic point group
Key words: Ferrotoroidic, Coset decomposition, Domain pairs

Introduction:

A ferroic crystal contains two or more equally stable domain states, volumes of the same homogeneous crystalline structure in different spatial orientations. These domains can coexist in a crystal and may be distinguished by the values of components of certain spontaneous macroscopic tensorial physical properties of the domains. Crystals, in which the domains may be distinguished by spontaneous magnetization, polarization and strain are called primary ferroic crystals (Newnham, 1974; Wadhawan, 2000) and are individually referred respectively as ferromagnetic, ferroelectric and ferroelastic ferroic crystals. The domains in these primary ferroic crystals can be switched by means of a magnetic field, an electric field, a mechanical stress or a combination of the three (Newnham & Cross, 1974) and consequently ferroic crystals are of technological importance for example, memory storage and electric and magnetic switches. A **fourth type** of primary ferroic crystals, a **ferrotoroidic** crystal, has been recently observed (Van Aken et al., 2007), where the domains are distinguished by a toroidal moment (Gorbatsevich et al., 1983; Schmid, 2001, 2003).

Keitsîro Aizu coined the collective term "Ferroic" for ferroelectrics, ferromagnetics and ferroelastics, having in common "Ferro", "-ics". Which can be switched, thus giving rise to hysteresis loops. Really akin are only ferroelectrics and ferromagnetic, which can be characterized by *a* single Shubnikov point group, where as ferroelastics with the symmetric 2^{nd} rank tensor spontaneous deformation have to be characterized by a pair of point groups , named "species". Based on the different terms of the density of stored free enthalpy, the "ferroic" nomenclature has been enlarged with a subdivision into "Primary", "Secondary" and "Tertiary" ferroics. Keeping in line with the ferroic nomenclature, it is now tempting to extend the primary ferroics with the introduction of the notion "ferrotoroidics", bearing a spontaneous toroidal moment $^{s}\mathbf{T}$. This has been done by using $^{s}\mathbf{T}$ as an order parameter with the nomenclature "Ferrotoroic".

The concept of toroidal moments in condensed-matter in physics and their long-range ordering is a ferrotoroidic. The ferrotoroidicity as a form of primary ferroic order can be understood both from microscopic (multipole expansion) and macroscopic (symmetry-based expansion of the free energy) points of view. The definition of the local toroidal moment and its transformation properties under the space-inversion and time reversal operations are highlighted.

Litvin S.Y.and Litvin D.B (1990) had tabulated the representative tensor pairs for all 32 point groups G, subgroups H and all physical property tensors T of rank 0, 1 and 2. This constitutes the basis for the sensorial classification of domain pairs in ferroic crystals which is given by using the group theoretical classification of the corresponding physical property tensor pairs. Litvin D.B calculated of the twin pairs for all the ferric spices taken 122 crystallographic groups as the prototypic point groups for the ferrotoroidic moments. N.V.S.S.Prabhakar (2007) calculated Ferromagnetic & Magnetoelectric polarizability domain pairs for the 32 grey groups as the prototypic point groups, Prof. S. Uma devi & M Vijaya Laxmi (2012) calculated ferrotoroidic domain pairs for 32 grey groups as a prototypic point groups. Here domain pairs of "Ferrotoroidic crystals" by using coset decomposition taking 32 point groups as a prototypic point groups are calculated.

COSET DECOMPOSITION:

Let H be a subgroup of a group G then the left (right) cosets of H in G provide a decomposition of G as a sum of mutually disjoint left (right) cosets,

i.e. G= $a_1 H + a_2 H +, ... + a_i H + ..., Where\ a_i \in$ G; i=1,2,3,.......... This is known as coset decomposition of G with respect to H. The arbitrary element a_i of each left-coset is called a representative of coset.

REPRESENTATIVEDOMAINPAIRS

Two domain states s_i and s_j form a domain pair (s_i, s_j) if $S_j = g_{ij} S_i$ and $S_i = g_{ij} S_j$ where g_{ij} is element of G. Two domain states that have different spontaneous toroidic vector are denoted as a ferrotoroidic domain pairs. A domain pair (s_i, s_j) is called a non ferrotoroidic domain pair if the two domain states have identical spontaneous toroidic vector (Janovec, 1972). Here we calculated domain pairs of "Ferrotoroidic crystals" by using coset decomposition taking 32 groups as prototypic point group.

FERROTOROIDIC DOMAIN PAIRS FOR FERROIC SPECIES 4/mmm F m

Consider the Ferroic species **4/mmm F m.** Where 4/mmm is a prototypic point group and **m** is a ferroic point group. The number of distinct domain pairs are 4. The coset decomposition of **4/mmm** with respect to the group **m** is given by

$$G = 4/mmm = E(E,\sigma_z) + C_{4z}^{+}(E,\sigma_z) + C_{4z}^{-}(E,\sigma_z) + I(E,\sigma_z) + C_{2a}(E,\sigma_z) + C_{2b}(E,\sigma_z) + \sigma_x(E,\sigma_z) + \sigma_x(E,\sigma_z).$$

The coset elements are $E,\ C_{4z}^{+},\ C_{4z}^{-},\ I,\ C_{2a},\ C_{2b},\ \sigma_x,\ \sigma_y$. the components of axial vector is represented by (T_1, T_2, T_3).

Let two domain states S_i and S_j Form a domain pairs (s_i, s_j) if $S_j = g_{ij} S_i$ and $S_i = g_{ij} S_j$, where g_{ij} is the element of G.

for example 2 : $\sigma_y = I\sigma_x = (-T_1, -T_2, 0)(-T_1, T_2, 0) = (T_1, -T_2, 0)$

$\sigma_x = I\sigma_y = (-T_1, -T_2, 0)(T_1, -T_2, 0) = (-T_1, T_2, 0)$

Hence (σ_x, σ_y) forms domain pairs. i.e. $(-T_1, T_2, 0)(T_1, -T_2, 0)$. Similarly the remaining domain pairs of G = **4/mmm F m** are in **Table 1.1**

Table 1.1

Domain pairs	Tensor Representatives
$(E,\ I)$	$(T_1, T_2, 0)\ (-T_1, -T_2, 0)$
$(C_{4z}^{+},\ C_{4z}^{-})$	$(-T_2, T_1, 0)\ (T_2, -T_1, 0)$
$(C_{2a},\ C_{2b})$	$(T_2, T_1, 0)\ (-T_2, -T_1, 0)$
$(\sigma_x,\ \sigma_y)$	$(-T_1, T_2, 0)\ (T_1, -T_2, 0)$

Similarly by taking the ferrotoroidic domain pairs for all 116 species are calculated using the above procedure. In **Table 1.2**, Domain pairs for all 116 ferrotoroidic species are given.

Table 1.2

S. No.	Groups (G)	Sub-Groups (H)	Ferrotorodic (aV)	Cosect elements	Domain Paris
1	1	1	(T_1, T_2, T_3)	E	$(T_1, T_2, T_3)\ (T_1, T_2, T_3)$
2	$\bar{1}$	1	(T_1, T_2, T_3)	(E, I)	$(T_1, T_2, T_3)\ (-T_1, -T_2, -T_3)$
3	2	1	(T_1, T_2, T_3)	(E, C_{2z})	$(T_1, T_2, T_3)\ (-T_1, -T_2, T_3)$
4	m	1	(T_1, T_2, T_3)	(E, σ_z)	$(T_1, T_2, T_3)\ (T_1, T_2, -T_3)$

5	2/m	1	(T_1,T_2,T_3)	(E,I), (C_{2z},σ_z)	(T_1,T_2,T_3) $(-T_1,-T_2,-T_3)$; $(-T_1,-T_2,T_3)$ $(T_1,T_2,-T_3)$.
		2	$(0,0,T_3)$	(E,I)	$(0,0,T_3)$ $(0,0,-T_3)$
		m	$(T_1,T_2,0)$	(E,I)	$(T_1,T_2,0)$ $(-T_1,-T_2,0)$
6	222	1	(T_1,T_2,T_3)	(E,C_{2z}), (C_{2x},C_{2y})	(T_1,T_2,T_3) $(-T_1,T_2,-T_3)$; $(T_1,-T_2,-T_3)$ $(-T_1,-T_2,T_3)$
		2	$(0,0,T_3)$	(E,C_{2x})	$(0,0,T_3)$ $(0,0,-T_3)$
7	mm2	1	(T_1,T_2,T_3)	(E, σ_x), (C_{2z},σ_y)	(T_1,T_2,T_3) $(-T_1,T_2,T_3)$; $(-T_1,-T_2,T_3)$ $(T_1,-T_2,T_3)$
		2	$(0,0,T_3)$	(E,σ_x)	$(0,0,T_3)$ $(0,0,T_3)$
		m	$(T_1,T_2,0)$	(E,σ_y)	$(T_1,T_2,0)$ $(T_1,-T_2,0)$
8	mmm	1	(T_1,T_2,T_3)	(E,I), (C_{2x},σ_x), (C_{2y},σ_y), (C_{2z},σ_z)	(T_1,T_2,T_3) $(-T_1,-T_2,-T_3)$; $(T_1,-T_2,-T_3)$ $(-T_1,T_2,T_3)$; $(-T_1,T_2,-T_3)$ $(T_1,-T_2,T_3)$; $(-T_1,-T_2,T_3)$ $(T_1,T_2,-T_3)$
		2	$(0,0,T_3)$	(E,I), (C_{2x},σ_x)	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$
		mm2	$(0,0,T_3)$	(E,I)	$(0,0,T_3)$ $(0,0,-T_3)$
9	4	1	(T_1,T_2,T_3)	(E,C_{4z}^{+}), (C_{4z}^{-},C_{2z})	(T_1,T_2,T_3) $(-T_2,T_1,T_3)$; $(T_2,-T_1,T_3)$ $(-T_1,-T_2,T_3)$
		2	$(0,0,T_3)$	(E,C_{4z}^{+})	$(0,0,T_3)$ $(0,0,T_3)$
10	$\bar{4}$	1	(T_1,T_2,T_3)	(E,C_{3z}), (S_{4z}^{-},S_{4z}^{+})	(T_1,T_2,T_3) $(-T_1,-T_2,T_3)$; $(T_2,-T_1,-T_3)$ $(-T_2,T_1,-T_3)$
		2	$(0,0,T_3)$	(E,S_{4z}^{-})	$(0,0,T_3)$ $(0,0,-T_3)$
11	4/m	1	(T_1,T_2,T_3)	(E,I), (C_{4z}^{+},S_{4z}^{-}), (C_{4z}^{-},S_{4z}^{+}), (C_{2z},σ_z)	(T_1,T_2,T_3) $(-T_1,-T_2,-T_3)$; $(-T_2,T_1,T_3)$ $(T_2,-T_1,-T_3)$; $(T_2,-T_1,T_3)$ $(-T_2,T_1,-T_3)$; $(-T_1,-T_2,T_3)$ $(T_1,T_2,-T_3)$
		2	$(0,0,T_3)$	(E,I), (C_{4z}^{-},C_{4z}^{+})	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,T_3)$

		m	$(T_1,T_2,0)$	(E,I), (C_{4z}^+,C_{4z}^-)	$(T_1,T_2,0)$ $(-T_1,-T_2,0)$; $(-T_2,T_1,0)$ $(T_2,-T_1,0)$
		4	$(0,0,T_3)$	(E,I)	$(0,0,T_3)$ $(0,0,-T_3)$
12	422	1	(T_1,T_2,T_3)	(E,C_{2x}), (C_{4z}^+,C_{4z}^-), (C_{2z},C_{2y}), (C_{2a},C_{2b})	(T_1,T_2,T_3) $(T_1,-T_2,-T_3)$; $(-T_2,T_1,T_3)$ $(T_2,-T_1,T_3)$; $(-T_1,T_2,-T_3)$ $(-T_1,-T_2,T_3)$; $(T_2,T_1,-T_3)$ $(-T_2,-T_1,-T_3)$
		2	$(0,0,T_3)$	(E,C_{2x}), (C_{4z}^+,C_{2b})	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$
		4	$(0,0,T_3)$	(E,C_{2x})	$(0,0,T_3)$ $(0,0,-T_3)$
13	4mm	1	(T_1,T_2,T_3)	(E,C_{2z}), (C_{4z}^+,C_{4z}^-), (σ_x,σ_y), $(\sigma_{da},\sigma_{db})$	(T_1,T_2,T_3) $(-T_1,-T_2,T_3)$; $(-T_2,T_1,T_3)$ $(T_2,-T_1,T_3)$; $(-T_1,T_2,T_3)$ $(T_1,-T_2,T_3)$; $(-T_2,-T_1,T_3)$ (T_2,T_1,T_3)
		2	$(0,0,T_3)$	(E,σ_x), (C_{4z}^+,σ_{da})	$(0,0,T_3)$ $(0,0,T_3)$; $(0,0,T_3)$ $(0,0,T_3)$
		m	$(T_1,T_2,0)$	(E,σ_y), (C_{4z}^+,C_{4z}^-)	$(T_1,T_2,0)$ $(T_1,-T_2,0)$; $(-T_2,T_1,0)$ $(T_2,-T_1,0)$
		mm2	$(0,0,T_3)$	(E,C_{4z}^+)	$(0,0,T_3)$ $(0,0,-T_3)$
		4	$(0,0,T_3)$	(E,σ_x)	$(0,0,T_3)$ $(0,0,-T_3)$
14	$\bar{4}2m$	1	(T_1,T_2,T_3)	(E,C_{2z}), (S_{4z}^+,S_{4z}^-), $(\sigma_{da},\sigma_{db})$, (C_{2x},C_{2y}),	(T_1,T_2,T_3) $(-T_1,-T_2,T_3)$; $(-T_2,T_1,-T_3)$ $(T_2,-T_1,-T_3)$; $(-T_2,-T_1,T_3)$ (T_2,T_1,T_3); $(T_1,-T_2,-T_3)$ $(-T_1,T_2,-T_3)$
		2	$(0,0,T_3)$	(E,C_{2x}), (S_{4z}^+,σ_{da})	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$
		m	$(T_1,T_2,0)$	(E,C_{2z}), (S_{4z}^+,S_{4z}^-)	$(T_1,T_2,0)$ $(-T_1,-T_2,0)$; $(-T_2,T_1,0)$ $(T_2,-T_1,0)$
		mm2	$(0,0,T_3)$	(E,C_{2x})	$(0,0,T_3)$ $(0,0,-T_3)$

15	4/mmm	1	(T_1,T_2,T_3)	(E,I), (C_{4z}^+,S_{4z}^-), (C_{4z}^-,S_{4z}^+), (C_{2x},σ_x), (C_{2y},σ_y), (C_{2z},σ_z), (C_{2a},σ_{da}), (C_{2b},σ_{db})	(T_1,T_2,T_3) $(-T_1,-T_2,-T_3)$; $(-T_2,T_1,T_3)$ $(T_2,-T_1,-T_3)$; $(T_2,-T_1,T_3)$ $(-T_2,T_1,-T_3)$; $(T_1,-T_2,-T_3)$ $(-T_1,T_2,T_3)$; $(-T_1,T_2,-T_3)$ $(T_1,-T_2,T_3)$; $(-T_1,-T_2,T_3)$ $(T_1,T_2,-T_3)$; $(T_2,T_1,-T_3)$ $(-T_2,-T_1,T_3)$; $(-T_2,-T_1,-T_3)$ (T_2,T_1,T_3)
		2	$(0,0,T_3)$	(E,I), (C_{4z}^+,S_{4z}^+), (σ_{da},C_{2a}), (σ_x,C_{2x})	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$
		m	$(T_1,T_2,0)$	(E,I), (C_{4z}^+,C_{4z}^-), (C_{2a},C_{2b}), (σ_x,σ_y)	$(T_1,T_2,0)$ $(-T_1,-T_2,0)$; $(-T_2,T_1,0)$ $(T_2,-T_1,0)$; $(T_2,T_1,0)$ $(-T_2,-T_1,0)$; $(-T_1,T_2,0)$ $(T_1,-T_2,0)$;
		mm2	$(0,0,T_3)$	(E,I), (C_{4z}^+,C_{2a})	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$
		4	$(0,0,T_3)$	(E,I), (σ_x,C_{2x})	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$
		4mm	$(0,0,T_3)$	(E,I)	$(0,0,T_3)$ $(0,0,-T_3)$
16	3	1	(T_1,T_2,T_3)	(C_3^+,C_3^-)	$(-T_2,T_1-T_2,T_3)$ $(-T_1+T_2,-T_1,T_3)$
17	$\overline{3}$	1	(T_1,T_2,T_3)	(E,I), (C_3^+,S_6^-), (C_3^-,S_6^+)	(T_1,T_2,T_3) $(-T_1,-T_2,-T_3)$; $(-T_2,T_1-T_2,T_3)$ $(T_2,-T_1+T_2,-T_3)$; $(-T_1+T_2,-T_1,T_3)$ $(T_1-T_2,T_1,-T_3)$
		3	$(0,0,T_3)$	(E,I)	$(0,0,T_3)$ $(0,0,-T_3)$
18	32	1	(T_1,T_2,T_3)	(E,C_{21}'), (C_{22}',C_3^+), (C_3^-,C_{23}')	(T_1,T_2,T_3) $(-T_1+T_2,T_2,-T_3)$; $(T_1,T_1-T_2,-T_3)$ $(-T_2,T_1-T_2,T_3)$; $(-T_1+T_2,-T_1,T_3)$ $(-T_2,-T_1,-T_3)$
		2	$(0,0,T_3)$	(C_3^+,C_3^-)	$(0,0,T_3)$ $(0,0,-T_3)$

		3	$(0,0,T_3)$	(E,C'_{21})	$(0,0,T_3)$ $(0,0,-T_3)$
19	3m	1	(T_1,T_2,T_3)	(E,σ_{d1}), (C_3^+,C_3^-), $(\sigma_{d2},\sigma_{d3})$	(T_1,T_2,T_3) $(T_1-T_2,-T_2,T_3)$; $(-T_2,T_1-T_2,T_3)$ $(-T_1+T_2,-T_1-T_3)$; $(-T_1,-T_1+T_2,T_3)$ (T_2,T_1,T_3)
		m	$(T_1,T_2,0)$	(C_3^+,C_3^-)	$(-T_2,T_1-T_2,0)$ $(-T_1+T_2,-T_1,0)$
		3	$(0,0,T_3)$	(E,σ_{d1})	$(0,0,T_3)$ $(0,0,-T_3)$
20	$\bar{3}m$	1	(T_1,T_2,T_3)	(E,I), (C_3^+,S_6^-), (C_3^-,S_6^+), (C'_{21},σ_{d1}), (C'_{22},σ_{d2}), (C'_{23},σ_{d3})	(T_1,T_2,T_3) $(-T_1,-T_2,-T_3)$; $(-T_2,T_1-T_2,T_3)$ $(T_2,-T_1+T_2,-T_3)$; $(-T_1+T_2,-T_1,T_3)$ $(T_1-T_2,T_1,-T_3)$; $(-T_1+T_2,T_2,-T_3)$ $(T_1-T_2,-T_2,T_3)$; $(T_1,T_1-T_2,-T_3)$ $(-T_1,-T_1+T_2,T_3)$; $(-T_2,-T_1,-T_3)$ (T_2,T_1,T_3)
		2	$(0,0,T_3)$	(E,σ_{d1}), (C_3^+,σ_{d2}), (C_3^-,σ_{d3})	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$
		m	(T_1,T_2,T_3)	(E,C'_{21}), (C_3^+,S_6^-), (C_3^-,S_6^+)	$(T_1,T_2,0)$ $(-T_1+T_2,T_2,0)$; $(-T_2,T_1-T_2,0)$ $(T_2,-T_1+T_2,0)$; $(-T_1+T_2-T_1,0)$ $(T_1-T_2,T_1,0)$
		3	$(0,0,T_3)$	(E,I), (σ_{d1},C'_{21})	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$
		3m	$(0,0,T_3)$	(E,I)	$(0,0,T_3)$ $(0,0,-T_3)$
21	6	1	(T_1,T_2,T_3)	(E,C_2), (C_6^-,C_3^+), (C_6^+,C_3^-)	(T_1,T_2,T_3) $(-T_1,-T_2,T_3)$; $(T_2,-T_1+T_2,T_3)$ $(-T_2,T_1-T_2,T_3)$; (T_1-T_2,T_1,T_3) $(-T_1+T_2,-T_1,T_3)$
		2	$(0,0,T_3)$	(C_6^+,C_6^-)	$(0,0,T_3)$ $(0,0,-T_3)$
		3	$(0,0,T_3)$	(E,C_2)	$(0,0,T_3)$ $(0,0,-T_3)$
22	$\bar{6}$	1	(T_1,T_2,T_3)	(E,σ_h), (S_3^-,C_3^-), (S_3^+,C_3^+)	(T_1,T_2,T_3) $(T_1,T_2,-T_3)$; $(-T_1+T_2,-T_1,-T_3)$ $(-T_1+T_2,-T_1,T_3)$; $(-T_2,T_1-T_2,-T_3)$ $(-T_2,T_1-T_2,T_3)$

		m	$(T_1,T_2,0)$	(S_3^-,S_3^+)	$(-T_1+T_2,-T_1,0)$ $(-T_2,T_1-T_2,0)$
		3	$(0,0,T_3)$	(E,σ_h)	$(0,0,T_3)$ $(0,0,-T_3)$
23	6/m	1	(T_1,T_2,T_3)	(E,I), (C_2,σ_h), (C_6^+,S_3^-), (C_6^-,S_3^+), (C_3^+,S_6^-), (C_3^-,S_6^+)	(T_1,T_2,T_3) $(-T_1,-T_2,-T_3)$; $(-T_1,-T_2,T_3)$ $(T_1,T_2,-T_3)$; (T_1-T_2,T_1,T_3) $(-T_1+T_2,-T_1,-T_3)$; $(T_2,-T_1+T_2,T_3)$ $(-T_2,T_1-T_2,-T_3)$; $(-T_2,T_1-T_2,T_3)$ $(T_2,-T_1+T_2,-T_3)$; $(-T_1+T_2,-T_1,T_3)$ $(T_1-T_2,T_1,-T_3)$
		2	$(0,0,T_3)$	(E,I), (C_6^+,S_3^-), (C_6^-,S_3^+)	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$
		m	$(T_1,T_2,0)$	(E,I), (C_6^+,S_3^-), (C_6^-,S_3^+)	$(T_1,T_2,0)$ $(-T_1,-T_2,0)$; $(T_1-T_2,T_1,0)$ $(-T_1+T_2,-T_1,0)$; $(T_2,-T_1+T_2,0)$ $(-T_2,T_1-T_2,0)$
		3	$(0,0,T_3)$	(E,I), (C_6^+,S_3^+)	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$
		6	$(0,0,T_3)$	(E,I)	$(0,0,T_3)$ $(0,0,-T_3)$
24	622	1	(T_1,T_2,T_3)	(E,C_2), (C_6^+,C_3^-), (C_6^-,C_3^+), (C'_{21},C''_{21}), (C'_{22},C''_{22}), (C'_{23},C''_{23})	(T_1,T_2,T_3) $(-T_1,-T_2,T_3)$; (T_1-T_2,T_1,T_3) $(-T_1+T_2,-T_1,T_3)$; $(T_2,-T_1+T_2,T_3)$ $(-T_2,T_1-T_2,T_3)$; $(-T_1+T_2,T_2,-T_3)$ $(T_1-T_2,-T_2,T_3)$; $(T_1,T_1-T_2,-T_3)$ $(-T_1,-T_1+T_2,-T_3)$; $(T_2,T_1,-T_3)$ $(-T_2,-T_1,-T_3)$
		2	$(0,0,T_3)$	(E,C_2), (C_6^+,C_3^-), (C_6^-,C_3^+)	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$
		3	$(0,0,T_3)$	(E,C'_{23}), (C_6^+,C''_{23})	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$
		6	$(0,0,T_3)$	(E,C'_{21})	$(0,0,T_3)$ $(0,0,-T_3)$

25	6mm	1	(T_1,T_2,T_3)	(E,C_2), (C_6^+,C_3^-), (C_6^-,C_3^+), $(\sigma_{d1},\sigma_{v1})$, $(\sigma_{d2}\sigma_{v2})$, $(\sigma_{d3},\sigma_{v3})$	(T_1,T_2,T_3) $(-T_1,-T_2,T_3)$; (T_1-T_2,T_1,T_3) $(-T_1+T_2,-T_1,T_3)$; $(T_2,-T_1+T_2,T_3)$ $(-T_2,T_1-T_2,T_3)$; $(T_1-T_2,-T_2,T_3)$ $(-T_1+T_2,T_2,T_3)$; $(-T_1,-T_1+T_2,T_3)$ (T_1,T_1-T_2,T_3); (T_2,T_1,T_3) $(-T_2,-T_1,T_3)$
		2	$(0,0,T_3)$	(E,σ_{d1}), (C_6^+,σ_{d2}), (C_6^-,σ_{d3})	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$
		m	$(T_1,T_2,0)$	(E,C_2), (C_6^+,C_3^-), (C_6^-,C_3^+)	$(T_1,T_2,0)$ $(-T_1,-T_2,0)$; $(T_1-T_2,T_1,0)$ $(-T_1+T_2,-T_1,0)$; $(T_2,-T_1+T_2,0)$ $(-T_2,T_1-T_2,0)$
		mm2	$(0,0,T_3)$	(C_6^+,C_6^-)	$(0,0,T_3)$ $(0,0,-T_3)$
		3	$(0,0,T_3)$	(E,σ_{d3}), (C_6^+,σ_{v3})	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$
		3m	$(0,0,T_3)$	(E,C_6^+)	$(0,0,T_3)$ $(0,0,-T_3)$
		6	$(0,0,T_3)$	(E,σ_{d3})	$(0,0,T_3)$ $(0,0,-T_3)$
26	$\bar{6}2m$	1	(T_1,T_2,T_3)	(E,σ_h), (S_3^-,C_3^-), (S_3^+,C_3^+), (C'_{21},σ_{v1}), (C'_{22},σ_{v2}), (C'_{23},σ_{v3})	(T_1,T_2,T_3) $(T_1,T_2,-T_3)$; $(-T_1+T_2,-T_1,-T_3)$ $(-T_1+T_2,-T_1,T_3)$; $(-T_2,T_1-T_2,-T_3)$ $(-T_2,T_1-T_2,T_3)$; $(-T_1+T_2,T_2,-T_3)$ $(-T_1+T_2,T_2,T_3)$; $(T_1,T_1-T_2,-T_3)$ (T_1,T_1-T_2,T_3); $(-T_2,-T_1,-T_3)$ $(-T_2,-T_1,T_3)$
		2	$(0,0,T_3)$	(E,σ_h), (C_3^-,S_3^-), (C_3^+,S_3^+)	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$
		m	$(T_1,T_2,0)$	(E,C'_{21}), (S_3^-,C'_{22}), (S_3^+,C'_{23})	$(T_1,T_2,0)$ $(-T_1+T_2,T_2,0)$; $(-T_1+T_2,-T_1,0)$ $(T_1,T_1-T_2,0)$; $(-T_2,T_1-T_2,0)$ $(-T_2,-T_1,0)$
		mm2	$(0,0,T_3)$	(S_3^-,S_3^+)	$(0,0,T_3)$ $(0,0,-T_3)$

		3	$(0,0,T_3)$	(E,S_3^-), (σ_{v3},C'_{23})	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$
		3m	$(0,0,T_3)$	(E,S_3^-)	$(0,0,T_3)$ $(0,0,-T_3)$
27	6/mmm	1	(T_1,T_2,T_3)	(E,I), (C_6^+,S_3^-), (C_6^-,S_3^+), (C_3^+,S_6^-), (C_3^-,S_6^+), (C_2,σ_h), (C'_{21},σ_{d1}), (C'_{22},σ_{d2}), (C'_{23},σ_{d3}), (C''_{21},σ_{v1}), (C''_{22},σ_{v2}), (C''_{23},σ_{v3})	(T_1,T_2,T_3) $(-T_1,-T_2,-T_3)$; (T_1-T_2,T_1,T_3) $(-T_1+T_2,-T_1,-T_3)$; $(T_2,-T_1+T_2,T_3)$ $(-T_2,T_1-T_2,-T_3)$; $(-T_2,T_1-T_2,T_3)$ $(T_2,-T_1+T_2,-T_3)$; $(-T_1+T_2,-T_1,T_3)$ $(T_1-T_2,T_1,-T_3)$; $(-T_1,-T_2,T_3)$ $(T_1,T_2,-T_3)$; $(-T_1+T_2,T_2,-T_3)$ $(T_1-T_2,-T_2,T_3)$; $(T_1,T_1-T_2,-T_3)$ $(-T_1,-T_1+T_2,T_3)$; $(-T_2,-T_1,-T_3)$ (T_2,T_1,T_3); $(T_1-T_2,-T_2,-T_3)$ $(-T_1+T_2,T_2,T_3)$; $(-T_1,-T_1+T_2,-T_3)$ (T_1,T_1-T_2,T_3); $(T_2,T_1,-T_3)$ $(-T_2,-T_1,T_3)$.
		2	$(0,0,T_3)$	(E,I), (C_6^+,S_3^-), (C_6^-,S_3^+), (σ_{d1},C'_{21}), (σ_{d2},C'_{22}), $(\sigma_{d3},C'_{23},)$	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$
		m	$(T_1,T_2,0)$	(E,I), (C_6^+,S_3^-), (C_6^-,S_3^+), (C'_{21},σ_{d1}), (C'_{22},σ_{d2}), (C'_{23},σ_{d3})	$(T_1,T_2,0)$ $(-T_1,-T_2,0)$; $(T_1-T_2,T_1,0)$ $(-T_1+T_2,-T_1,0)$; $(T_2,-T_1+T_2,0)$ $(-T_2,T_1-T_2,0)$; $(-T_1+T_2,T_2,0)$ $(T_1-T_2,-T_2,0)$; $(T_1,T_1-T_2,0)$ $(-T_1,-T_1+T_2,0)$; $(-T_2,-T_1,0)$ $(T_2,T_1,0)$
		mm2	$(0,0,T_3)$	(E,I), (C_6^+,S_3^-), (C_6^-,S_3^+)	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$

		3	$(0,0,T_3)$	(E,I), (C_6^+,σ_h), (σ_{d3},C'_{23}), (σ_{v3},C''_{23})	$(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$
		3m	$(0,0,T_3)$	$(E,I),(C_6^+,S_3^-)$	$(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$
		6	$(0,0,T_3)$	$(E,I),(C'_{21},\sigma_{d1})$	$(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$
		6mm	$(0,0,T_3)$	(E,I)	$(0,0,T_3)\ (0,0,-T_3)$
28	23	1	(T_1,T_2,T_3)	(E,C_{2x}), (C_{2y},C_{2z}), (C_{31}^+,C_{34}^+), (C_{32}^+,C_{33}^+), (C_{31}^-,C_{32}^-), (C_{33}^-,C_{34}^-)	$(T_1,T_2,T_3)\ (T_1,-T_2,-T_3)$ $(-T_1,T_2,-T_3)\ (-T_1,-T_2,T_3)$ $(T_3,T_1,T_2)\ (T_3,-T_1,-T_2)$ $(-T_3,T_1,-T_2)\ (-T_3,-T_1,T_2)$ $(T_2,T_3,T_1)\ (T_2,-T_3,-T_1)$ $(-T_2,T_3,-T_1)\ (-T_2,-T_3,T_1)$
		2	$(0,0,T_3)$	(E,C_{2x}), (C_{31}^+,C_{32}^+), (C_{31}^-,C_{32}^-)	$(0,0,T_3)\ (0,0,-T_3)$; $(T_3,0,0)\ (-T_3,0,0)$; $(0,T_3,0)\ (0,-T_3,0)$
		3	$(0,0,T_3)$	(E,C_{2x}), (C_{2y},C_{2z})	$(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$
29	m3	1	(T_1,T_2,T_3)	(E,I), (C_{2x},σ_x), (C_{2y},σ_y), (C_{2z},σ_z), (C_{31}^+,S_{61}^-), (C_{32}^+,S_{62}^-), (C_{33}^+,S_{63}^-), (C_{34}^+,S_{64}^-), (C_{31}^-,S_{61}^+), (C_{32}^-,S_{62}^+), (C_{33}^-,S_{63}^+), (C_{34}^-,S_{64}^+)	$(T_1,T_2,T_3)\ (-T_1,-T_2,-T_3)$; $(T_1,-T_2,-T_3)\ (-T_1,T_2,T_3)$; $(-T_1,T_2,-T_3)\ (T_1,-T_2,T_3)$; $(-T_1,-T_2,T_3)\ (T_1,T_2,-T_3)$; $(T_3,T_1,T_2)\ (-T_3,-T_1,-T_2)$; $(-T_3,T_1,-T_2)\ (T_3,-T_1,T_2)$; $(-T_3,-T_1,T_2)\ (T_3,T_1,-T_2)$; $(T_3,-T_1,-T_2)\ (-T_3,T_1,T_2)$; $(T_2,T_3,T_1)\ (-T_2,-T_3,-T_1)$; $(T_2,-T_3,-T_1)\ (-T_2,T_3,T_1)$; $(-T_2,T_3,-T_1)\ (T_2,-T_3,T_1)$;

					$(-T_2,-T_3,T_1)$ $(T_2,T_3,-T_1)$
		2	$(0,0,T_3)$	(E,I), (σ_x,C_{2x}), (C_{31}^+,S_{61}^-), (S_{62}^-,C_{32}^+), (C_{31}^-,S_{61}^+), (S_{62}^+,C_{32}^-)	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$; $(T_3,0,0)$ $(-T_3,0,0)$; $(T_3,0,0)$ $(-T_3,0,0)$; $(0,T_3,0)$ $(0,-T_3,0)$; $(0,T_3,0)$ $(0,-T_3,0)$
		m	$(T_1,T_2,0)$	(E,C_{2z}), (C_{2x},C_{2y}), (C_{31}^+,C_{34}^+), (C_{32}^+,C_{33}^+), (C_{31}^-,C_{33}^-), (C_{32}^-,C_{34}^-)	$(T_1,T_2,0)$ $(-T_1,-T_2,0)$; $(T_1,-T_2,0)$ $(-T_1,T_2,0)$; $(0,T_1,T_2)$ $(0,-T_1,-T_2)$; $(0,T_1,-T_2)$ $(0,-T_1,T_2)$; $(T_2,0,T_1)$ $(-T_2,0,-T_1)$; $(T_2,0,-T_1)$ $(-T_2,0,T_1)$
		mm2	$(0,0,T_3)$	(E,I), (C_{31}^+,C_{32}^+), (C_{31}^-,C_{32}^-)	$(0,0,T_3)$ $(0,0,-T_3)$; $(T_3,0,0)$ $(-T_3,0,0)$; $(0,T_3,0)$ $(0,-T_3,0)$
		3	$(0,0,T_3)$	(E,I), (σ_x,C_{2x}), (σ_y,C_{2y}), (C_{2z},σ_z)	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$ $(0,0,T_3)$ $(0,0,-T_3)$

30	432	1	(T_1,T_2,T_3)	(E,C_{2x}), (C_{2y},C_{2z}), (C^+_{31},C^+_{34}), (C^+_{32},C^+_{33}) (C^-_{31},C^-_{32}), $(C^-_{33},C^-_{34},)$ (C_{2a},C^-_{4z}), (C_{2b},C^+_{4z}), (C_{2c},C^+_{4y}), (C_{2d},C_{2f}), (C_{2e},C^-_{4y}), (C^+_{4x},C^-_{4x})	(T_1,T_2,T_3) $(T_1,-T_2,-T_3)$; $(-T_1,T_2,-T_3)$ $(-T_1,-T_2,T_3)$; (T_3,T_1,T_2) $(T_3,-T_1,-T_2)$; $(-T_3,T_1,-T_2)$ $(-T_3,-T_1,T_2)$; (T_2,T_3,T_1) $(T_2,-T_3,-T_1)$; $(-T_2,T_3,-T_1)$ $(-T_2,-T_3,T_1)$; $(T_2,T_1,-T_3)$ $(T_2,-T_1,T_3)$; $(-T_2,-T_1,-T_3)$ $(-T_2,T_1,T_3)$; $(T_3,-T_2,T_1)$ $(T_3,T_2,-T_1)$; $(-T_1,T_3,T_2)$ $(-T_1,-T_3,-T_2)$; $(-T_3,-T_2,-T_1)$ $(-T_3,T_2,T_1)$; $(T_1,-T_3,T_2)$ $(T_1,T_3,-T_2)$
		2	$(0,0,T_3)$	(E,C_{2x}), (C^+_{4z},C_{2a}), (C^+_{31},C^+_{32}), (C_{2c},C_{2e}), (C^-_{31},C^-_{32}), (C_{2d},C_{2f})	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$; $(T_3,0,0)$ $(-T_3,0,0)$; $(T_3,0,0)$ $(-T_3,0,0)$; $(0,T_3,0)$ $(0,-T_3,0)$; $(0,T_3,0)$ $(0,-T_3,0)$
		4	$(0,0,T_3)$	(E,C_{2x}), $(C_{2c},C_{2e},)$, (C_{2d},C_{2f})	$(0,0,T_3)$ $(0,0,-T_3)$; $(T_3,0,0)$ $(-T_3,0,0)$; $(0,T_3,0)$ $(0,-T_3,0)$
		3	$(0,0,T_3)$	(E,C_{2x}), (C_{2z},C_{2y}), (C_{2a},C_{2b}), (C_{2c},C_{2d})	$(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$; $(0,0,T_3)$ $(0,0,-T_3)$

31	$\bar{4}3m$	1	(T_1,T_2,T_3)	(E,C_{2x}), (C_{2y},C_{2z}), (C_{31}^{+},C_{34}^{+}), (C_{32}^{+},C_{33}^{+}), (C_{31}^{-},C_{32}^{-}), (C_{33}^{-},C_{34}^{-}), (σ_{da},S_{4z}^{+}), (σ_{db},S_{4z}^{-}), (σ_{dc},S_{4y}^{-}), $(\sigma_{dd},\sigma_{df})$, (σ_{de},S_{4y}^{+}), (S_{4x}^{-},S_{4x}^{+})	$(T_1,T_2,T_3)\ (T_1,-T_2,-T_3)$; $(-T_1,T_2,-T_3)\ (-T_1,-T_2,T_3)$; $(T_3,T_1,T_2)\ (T_3,-T_1,-T_2)$; $(-T_3,T_1,-T_2)\ (-T_3,-T_1,T_2)$; $(T_2,T_3,T_1)\ (T_2,-T_3,-T_1)$; $(-T_2,T_3,-T_1)\ (-T_2,-T_3,T_1)$; $(-T_2,-T_1,T_3)\ (-T_2,T_1,-T_3)$; $(T_2,T_1,T_3)\ (T_2,-T_1,-T_3)$; $(-T_3,T_2,-T_1)\ (-T_3,-T_2,T_1)$; $(T_1,-T_3,-T_2)\ (T_1,T_3,T_2)$; $(T_3,T_2,T_1)\ (T_3,-T_2,-T_1)$; $(-T_1,T_3,-T_2)\ (-T_1,-T_3,T_2)$
		2	$(0,0,T_3)$	(E,C_{2x}), $(\sigma_{da},S_{4z}^{+},)$, (C_{31}^{+},C_{32}^{+}), $(\sigma_{de},\sigma_{dc})$, (C_{31}^{-},C_{32}^{-}), $(\sigma_{df},\sigma_{dd})$	$(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$; $(T_3,0,0)\ (-T_3,0,0)$; $(T_3,0,0)\ (-T_3,0,0)$; $(0,T_3,0)\ (0,-T_3,0)$; $(0,T_3,0)\ (0,-T_3,0)$
		m	$(T_1,T_2,0)$	(E,C_{2z}), (C_{2x},C_{2y}), $({}_{31}^{+},C_{34}^{+})$, (C_{32}^{+},C_{33}^{+}), $(\sigma_{dc},\sigma_{df})$, (C_{31}^{-},C_{32}^{-})	$(T_1,T_2,0)\ (-T_1,-T_2,0)$; $(T_1,-T_2,0)\ (-T_1,T_2,0)$; $(0,T_1,T_2)\ (0,-T_1,-T_2)$; $(0,T_1,-T_2)\ (0,-T_1,T_2)$; $(0,T_2,-T_1)\ (0,-T_2,-T_1)$; $(T_2,0,T_1)\ (T_2,0,-T_1)$
		mm2	$(0,0,T_3)$	(E,C_{2x}), (C_{31}^{+},C_{32}^{+}), (C_{31}^{-},C_{32}^{-})	$(0,0,T_3)\ (0,0,-T_3)$; $(T_3,0,0)\ (-T_3,0,0)$; $(0,T_3,0)\ (0,-T_3,0)$
		3	$(0,0,T_3)$	(E,C_{2x}), (C_{2z},C_{2y}), $(\sigma_{da},\sigma_{db})$, $(\sigma_{dc},\sigma_{dd})$	$(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$

		3m	$(0,0,T_3)$	(E,C_{2x}), (C_{2y},C_{2z})	$(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$
32	m3m	1	(T_1,T_2,T_3)	(E,I), (C_{2x},σ_x), (C_{2y},σ_y), (C_{2z},σ_z), (C_{31}^+,S_{61}^-), (C_{32}^+,S_{62}^-), (C_{33}^+,S_{63}^-), (C_{34}^+,S_{64}^-), (C_{31}^-,S_{61}^+), (C_{32}^-,S_{62}^+), (C_{33}^-,S_{63}^+), (C_{34}^-,S_{64}^+), (C_{2a},σ_{da}), (C_{2b},σ_{db}), (C_{2c},σ_{dc}), (C_{2d},σ_{dd}), (C_{2e},σ_{de}), (C_{2f},σ_{df}), (C_{4x}^+,S_{4x}^-), (C_{4y}^+,S_{4y}^-), (C_{4z}^+,S_{4z}^-), (C_{4x}^-,S_{4x}^+), (C_{4y}^-,S_{4y}^+), (C_{4z}^-,S_{4z}^+)	$(T_1,T_2,T_3)\ (-T_1,-T_2,-T_3)$; $(T_1,-T_2,-T_3)\ (-T_1,T_2,T_3)$; $(-T_1,T_2,-T_3)\ (T_1,-T_2,T_3)$; $(-T_1,-T_2,T_3)\ (T_1,T_2,-T_3)$; $(T_3,T_1,T_2)\ (-T_3,-T_1,-T_2)$; $(-T_3,T_1,-T_2)\ (T_3,-T_1,T_2)$; $(-T_3,-T_1,T_2)\ (T_3,T_1,-T_2)$; $(T_3,-T_1,-T_2)\ (-T_3,T_1,T_2)$; $(T_2,T_3,T_1)\ (-T_2,-T_3,-T_1)$; $(T_2,-T_3,-T_1)\ (-T_2,T_3,T_1)$; $(-T_2,T_3,-T_1)\ (T_2,-T_3,T_1)$; $(-T_2,-T_3,T_1)\ (T_2,T_3,-T_1)$; $(T_2,T_1,-T_3)\ (-T_2,-T_1,T_3)$; $(-T_2,-T_1,-T_3)\ (T_2,T_1,T_3)$; $(T_3,-T_2,T_1)\ (-T_3,T_2,-T_1)$; $(-T_3,-T_2,-T_1)\ (T_3,T_2,T_1)$; $(-T_1,T_3,T_2)\ (T_1,-T_3,T_2)$; $(-T_1,-T_3,-T_2)\ (T_1,T_3,T_2)$; $(T_1,-T_3,T_2)\ (-T_1,T_3,-T_2)$; $(T_3,T_2,-T_1)\ (-T_3,-T_2,T_1)$; $(T_1,T_3,-T_2)\ (-T_1,-T_3,T_2)$; $(-T_3,T_2,T_1)\ (T_3,-T_2,-T_1)$; $(T_2,-T_1,T_3)\ (-T_2,T_1,-T_3)$

		2	$(0,0,T_3)$	(E,I), (σ_x,C_{2x}), (σ_{da},C_{2a}), (C_{4z}^+,S_{4z}^-), (C_{31}^+,S_{61}^-), (S_{62}^-,C_{32}^+), (C_{2c},σ_{dc}), (σ_{de},C_{2e}), (C_{31}^-,S_{61}^+), (S_{62}^+,C_{32}^-), (C_{2d},σ_{dd}), (σ_{df},C_{2f})	$(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$; $(T_3,0,0)\ (-T_3,0,0)$; $(T_3,0,0)\ (-T_3,0,0)$; $(T_3,0,0)\ (-T_3,0,0)$; $(T_3,0,0)\ (-T_3,0,0)$; $(0,T_3,0)\ (0,-T_3,0)$; $(0,T_3,0)\ (0,-T_3,0)$ $(0,T_3,0)\ (0,-T_3,0)$; $(0,T_3,0)\ (0,-T_3,0)$
		m	$(T_1,T_2,0)$	(E,I), (C_{2x},σ_x), (C_{31}^+,S_{61}^-), (C_{32}^+,S_{62}^-), (C_{31}^-,S_{61}^+), (C_{32}^-,S_{62}^+), (C_{2a},σ_{da}), (C_{2c},σ_{dc}), (C_{2d},σ_{dd}), (C_{2e},σ_{de}), (C_{2f},σ_{df}), (C_{4z}^+,S_{4z}^-)	$(T_1,T_2,0)\ (-T_1,-T_2,0)$; $(T_1,-T_2,0)\ (-T_1,T_2,0)$; $(0,T_1,T_2)\ (0,-T_1,-T_2)$; $(0,T_1,-T_2)\ (0,-T_1,T_2)$; $(T_2,0,T_1)\ (-T_2,0,-T_1)$; $(T_2,0,-T_1)\ (-T_2,0,T_1)$; $(T_2,T_1,0)\ (-T_2,-T_1,0)$; $(0,-T_2,T_1)\ (0,T_2,-T_1)$; $(-T_1,0,T_2)\ (T_1,0,-T_2)$; $(0,-T_2,-T_1)\ (0,T_2,T_1)$; $(-T_1,0,-T_2)\ (T_1,0,T_2)$; $(-T_2,T_1,0)\ (T_2,-T_1,0)$;
		mm2	$(0,0,T_3)$	(E,C_{2x}), (C_{4z}^+,C_{2a}), (C_{31}^+,C_{32}^+), (C_{2c},C_{2e}), (C_{31}^-,C_{32}^-), (C_{2d},C_{2f})	$(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$; $(T_3,0,0)\ (-T_3,0,0)$; $(T_3,0,0)\ (-T_3,0,0)$; $(0,T_3,0)\ (0,-T_3,0)$; $(0,T_3,0)\ (0,-T_3,0)$

		4	$(0,0,T_3)$	(E,I), (C_{2x},σ_x), (C_{31}^+,S_{61}^-), (S_{62}^-,C_{32}^+), (C_{31}^-,S_{61}^+), (S_{62}^+,C_{32}^-)	$(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$; $(T_3,0,0)\ (-T_3,0,0)$; $(T_3,0,0)\ (-T_3,0,0)$; $(0,T_3,0)\ (0,-T_3,0)$; $(0,T_3,0)\ (0,-T_3,0)$
		4mm	$(0,0,T_3)$	(E,C_{2x}), (C_{31}^+,C_{32}^+), (C_{31}^-,C_{32}^-)	$(0,0,T_3)\ (0,0,-T_3)$; $(T_3,0,0)\ (-T_3,0,0)$; $(0,T_3,0)\ (0,-T_3,0)$
		3	$(0,0,T_3)$	(E,I), (σ_x,C_{2x}), (σ_y,C_{2y}), (C_{2z},σ_z), (C_{2a},σ_{da}), (C_{2b},σ_{db}), (C_{2c},σ_{dc}), (C_{2d},σ_{dd})	$(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$; $(0,0,T_3)\ (0,0,-T_3)$; $(T_3,0,0)\ (-T_3,0,0)$; $(0,T_3,0)\ (0,-T_3,0)$;
		3m	$(0,0,T_3)$	(E,I), (S_{62}^-,C_{32}^+), (S_{63}^-,C_{33}^+), (C_{34}^+,S_{64}^-)	$(0,0,T_3)\ (0,0,-T_3)$; $(T_3,0,0)\ (-T_3,0,0)$; $(T_3,0,0)\ (-T_3,0,0)$; $(T_3,0,0)\ (-T_3,0,0)$

Conclusions:

In this paper, Ferrotoroidic domain pairs are calculated by using coset decomposition for **116 ferroic species**, taking 32 point groups as prototypic point group. As an example the ferroic species **4/mmm F m** is illustrated. This can be extended for calculating the domain pains for all 116 ferroic species, taking 32 point groups as prototypic point groups. And the results are given the **table 1.2.**

References:

1. D. B. Litvin. (2008) “Feroic classifications extended to Ferrotoroidic Crystals”. Acta Cryst.(2008). A64, 316-320.
2. Litvin, Daniel B. (2008). “Property Tensors and Ferrotoroidic Domains”. Ferroelectrics, 376:1, 158-167.
3. H. Schmid. (2001). “On Ferrotoroidic and electrotoroidic, magnetotoroidic and piezotoroidic effects”. Ferroelectrics, 252, 41-50.
4. A.A. Gorbasesvich, Yu8.V.Kopaev, and V.V.Tugushev, “Anomalous nonlineara effects aat phase transitions to ferroelectric and magnetoelectric states.” Zh.Eksp.Teor. Fiz.85, 1107-1121(Sov.Phys.JETP 58, 643-651(1983).
5. N.A.Spaldin, M.Fiebig, M.Mostovoy.(2008). “The toroidic momen in condensed. Matter Physics and its Relation to the Magnetoelectric Effect”. J.Phys. Condens. Maataeara 20, 434203.
6. V.Janovec.(1972). “Group analysis of domains and domain pairs”. J. Phys. 22, 974-994.

7. N.V.S.S. Prabhakar.(2007). “Ferromagnetic &maaganetoaealeacataric Polariazability domain pairs”. M.Phil. Dissertation, Andhra University.
8. Prof.S.Umadevi, M.Vijaya Laxmi.(2012). “Ferrotoroidic Property or ferroic species”. IJERA, ISSN: 2248-9622, Vol.2, Issue 1,pp 896-903.
9. Bhagavantham, S.(1966). “Crystal Symmetry and physical properties”. Academic Press, London.
10. Bradley, C.J. and Craacknaell, A.P.(1972). “The Mathematical Theory of Symmetry in Solidss”, Clarendon Pressw, Oxford.
11. Litvin, DB, V.Janovec, E.Dvovakova, and T.R.Wike. (1989). Acta Cryst. A 45, 801-802.-
12. Litvin, DB, Litvin S.Y. “Rank of 0,1,2,3 magnetic and Non-magnetic physical property tensors”. Acta Cryst. A47, 290-292(1991).
13. Janovec.V, (1972). Czech.J.Phys. B22, p974.
14. Janovec.V, Litvin, DB.(2006). Aacta cryst. A62,98-102.
15. Dubovik, V.M.&Tugushev, V.V.”Toroidic moments in electrodynamics and solid-state physics”. Phys.Rep 187, 145-202.(1990).
16. Litvin, DB, (2010).”Ferroic classification extended to Ferotoroidic crystals”. Supplementary material. The Pennsylvania state University, Penn State Books, PA 19610-6009.
17. Litvin, DB. (2010). “Tenso Component distinction of magnetic, non-ferroelastic domain pairs”. Supplementary material.
18. Van Aken, B.B, Rivera, A..P., Schmid, H. & Fiebig, M. (2007). Observation of ferrotoroidic dsomains.