Sensitivity analysis of supply and demand in a fully interval transportation problem

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Abstract
In this paper, sensitivity analysis of supply and demand parameters of an interval transportation problem is presented. A new solution method namely, upper-lower method is proposed to determine the ranges of supply and demand parameters in an interval transportation problem such that its optimal basis is invariant. To illustrate the proposed method a numerical example is solved. Then, the upper-lower method is extended to fuzzy transportation problems. The study of sensitivity by the proposed method can be served as an important tool for the decision makers for taking appropriate decision when they are handling various types of logistic problems having imprecise parameters.

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1.Introduction
The transportation problem is one of the earliest applications of linear programming problems. Transportation models play an important role in logistics and supply-chain management for reducing cost and improving service. Some previous studies have devised solution procedure for the transportation problem with precise supply and demand parameters. Efficient algorithms have been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly. In real world applications, the supply and demand quantities in the transportation problem are sometimes hardly specified precisely because of changing economic conditions. Hence in order to reduce information costs and also to construct a real model, the use of interval and fuzzy transportation problems are more appropriate. Interval transportation problem is a relevant tool to address the intrinsic uncertainty in models of real-world problems. In an interval (or fuzzy) transportation problem, information about the range of variation of some (or all) of the parameters is available, which allows to specify a model with intervals (or fuzzy numbers). Interval and fuzzy transportation problems have been studied by various authors [9, 15, 18, 19, 20].

Now a days, most of research papers are concentrated how to solve transportation problems efficiently, but only few papers are focused on sensitivity analysis. Sensitivity analysis is one of the most interesting and preoccupying areas in optimization. Many attempts are made to investigate the problem's behavior when the input data changes. Usually, variation occurs in the right hand side of the constraints and/or the objective function coefficients. Sensitivity analysis is to analyze the effect of the changes of the objective function coefficients and the effect of changes of the right hand side constraints on the optimal value of the objective function as well as the validity ranges of these effects. Most of sensitivity analysis of a transportation problem is based on the assumption of optimal solution of a transportation problem. Gal [10] discussed post optimality analysis, parametric programming and related topics. Srinivasan and Thompson [21], Intrator and Paroush [12] and Arsham [2] studied the conventional sensitivity analysis of a transportation problem and some interesting results were derived. Adlakha and Arsham [1] proposed a pivotal algorithm for dealing with sensitivity analysis of a transportation problem without using any extra variables. Intrator and Engelberg [13] considered sensitivity analysis of a transportation problem by reducing the dimensionality of associated tableau. Doustdargholi et al. [8] studied the sensitivity analysis of right-hand-side parameter in a transportation problem. Badra [3] introduced sensitivity analysis of multiobjective transportation problems. Kang-Ting Ma and Ue-Pyng Wen [14] presented support set invariant sensitivity analysis in a degenerate transportation problem. Chi-Jen Lin and Ue-Pyng Wen [4,5] studied sensitivity analysis of an assignment problem. Lucia Cabulea [16] discussed the sensitivity analysis of costs in a transportation problem. Chi-Jen Lin [7] has presented two types of sensitivity range for an assignment problem. Chi-Jen Lin et al.[6] have studied three types of sensitivity analysis of a fuzzy assignment problem using labeling algorithm.

In this paper, we propose a new method namely, upper-lower method to develop the sensitivity analysis of supply and demand parameters in an interval transportation problem. So, we will show that the basis to the interval transportation problem remains optimal when the interval supply and demand vary between the interval limits. An illustrative example is presented to clarify the idea of the upper-lower method. Then, we extend the proposed method to fully fuzzy transportation problems. The proposed method provides an applicable information which helps the decision makers while they are handling various types of logistic problems having imprecise parameters.

2. Preliminaries

We need the following definitions of the basic arithmetic operators and partial ordering on closed bounded intervals which can be found in [11,17].

Let D denote the set of all closed bounded intervals on the real line R.
That is, $D = \{[a,b], a \le b$ and a and b are in R$\}$.

Definition 2.1: Let $A = [a,b]$ and $B = [c,d]$ be in D. Then,

(i) $A \oplus B = [a+c, b+d]$;

(ii) $A \Theta B = [a-d, b-c]$;

(iii) $kA = [ka, kb]$ if k is a positive real number ;

(iv) $kA = [kb, ka]$ if k is a negative real number and

(v) $A \otimes B = [p,q]$ where $p = \min.\{ac, ad, bc, bd\}$ and $q = \max.\{ac, ad, bc, bd\}$.

Definition 2.2: Let $A = [a,b]$ and $B = [c,d]$ be in D. Then,

(i) $A \le B$ if $a \le c$ and $b \le d$;

(ii) $A \ge B$ if $B \le A$, that is, $a \ge c$ and $b \ge d$ and

(iii) $A = B$ if $A \le B$ and $B \le A$, that is, $a = c$ and $b = d$.

3. Fully Interval Integer Transportation Problem

Consider the following fully interval integer transportation problem (IP):

(IP) Minimize $[z_1, z_2] = \sum_{i=1}^{m} \sum_{j=1}^{n} [c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}]$

subject to

$$\sum_{j=1}^{n} [x_{ij}, y_{ij}] = [a_i, p_i], \ i = 1,2,...,m \qquad (1)$$

$$\sum_{i=1}^{m} [x_{ij}, y_{ij}] = [b_j, q_j], \ j = 1,2,...,n \qquad (2)$$

$$x_{ij} \ge 0, \ y_{ij} \ge 0, \ i = 1,2,...,m \text{ and } j = 1,2,...,n \text{ and are integers} \qquad (3)$$

where c_{ij} and d_{ij} are positive real numbers for all i and j , a_i and p_i are positive real numbers for all i and b_j and q_j are positive real numbers for all j.

Definition 3.1: The set $\{[x_{ij}, y_{ij}]$, for all $i = 1,2,...,m$ and $j = 1,2,...,n\}$ is said to be a feasible solution of (IP) if they satisfy the equations (1), (2) and (3).

Definition 3.2: A feasible solution $\{[x_{ij}, y_{ij}]$, for $i = 1,2,...,m$ and $j = 1,2,...,n\}$ of the problem (IP) is said to be an optimal solution of (IP) if

$$\sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}] \leq \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij}, d_{ij}] \otimes [u_{ij}, v_{ij}],$$

for $i = 1,2,...,m$ and $j = 1,2,...,n$ and for all feasible $\{[u_{ij}, v_{ij}]$ for $i = 1,2,...,m$ and $j = 1,2,...,n\}$.

Now, we need the following theorem which finds a relation between optimal solutions of a fully interval integer transportation problem and a pair of induced transportation problems and also, is used in the proposed method which can be found in Pandian and Natarajan [18].

Theorem 3.1: If the set $\{y^{\circ}_{ij}$, for all i and j $\}$ is an optimal solution of the upper bound transportation problem (UP) of the problem (IP) where

(UP) Minimize $z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}y_{ij}$

subject to

$$\sum_{j=1}^{n} y_{ij} = p_i,\ i = 1,2,...,m$$

$$\sum_{i=1}^{m} y_{ij} = q_j,\ j = 1,2,...,n$$

$y_{ij} \geq 0$, $i = 1,2,...,m$ and $j = 1,2,...,n$ and are integers

and the set $\{x^{\circ}_{ij}$, for all i and j $\}$ is an optimal solution of the lower bound transportation problem (LP) of the problem (IP) where

(LP) Minimize $z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1,2,...,m$$

$$\sum_{i=1}^{m} x_{ij} = b_j,\ j = 1,2,...,n$$

$x_{ij} \geq 0$, $i = 1,2,...,m$ and $j = 1,2,...,n$ and are integers,

then the set of intervals $\{[x^{\circ}_{ij}, y^{\circ}_{ij}]$, for all i and j $\}$ is an optimal solution of the problem (IP) provided $x^{\circ}_{ij} \leq y^{\circ}_{ij}$, for all i and j.

4. Sensitivity Analysis

Sensitivity analysis is used to determine how "sensitive" a model is to change in the value of the parameters of the model and to change in the structure of the model. Parameter sensitivity allows decision makers to determine what level of accuracy is necessary for a parameter to make the model sufficiently useful and valid. By showing how the model behavior responds to changes in parameter values, sensitivity analysis is a useful tool in model building as well as in model evaluation and it can also indicate which parameter values are reasonable to use in the model.

4.1. Sensitivity Analysis for Interval supply and demand parameters

Consider the sensitivity analysis of an interval supply and demand in an interval transportation problem. Observe that this change maintains a balanced interval transportation problem. As per the Theorem 3.1., for analyzing the sensitivity analysis of supply and demand parameters of an interval transportation problem, we enough to do the sensitivity analysis of supply and demand parameters of upper bound transportation problem (UP) and the lower bound transportation problem (LP) of the interval transportation problem (IP).

Now, to analyze the sensitivity of the supply and demand values in the problem (UP), we consider the allotment table for the upper bound transportation problem obtained by the zero point method. Let $p_t = \max.\{p_1, p_2, ..., p_m\}$ and $q_k = \max.\{q_1, q_2, ..., q_n\}$. Now, we replace p_i by $p_i + \alpha_i$, i=1,2,..., m; $i \neq t$ and p_t by $p_t - \sum_{\substack{i=i \\ i \neq t}}^{m} \alpha_i$ and also, we replace q_j by $q_j + \lambda_j$, j=1,2,...,n; $j \neq k$ and q_k by $q_k - \sum_{\substack{j=i \\ j \neq k}}^{n} \lambda_j$ We compute the values of α_i, λ_j, for all i and j such that the optimal basis is invariant. That is, the allotment conditions of the zero point method should be satisfied.
Therefore, using allotment conditions of the zero point method, we can compute the ranges of each p_i, q_j, for all i and j, to maintain the current optimal basis .

Similarly, we can determine the ranges of all supply and demand value in the problem (LP) to maintain the current optimal basis.

Finally, we obtain the ranges of all supply and demand values in the interval transportation problems to maintain the current optimal basis by joining the results of the problems (LP) and (UP).

4.2. Upper-Lower Method

We, now introduce a new method namely, upper-lower method to study the sensitivity analysis of supply and demand in interval transportation problems.

The upper-lower method proceeds as follows.

Step 1. Construct the upper bound and lower bound transportation problems for the given interval transportation problem.

Step 2. Determine the optimal solution of the upper bound interval transportation problem. Say, $\{y_{ij}^{\circ}, \mathrm{i} = 1,2,...,m \text{ and } \mathrm{j} = 1,2,...,\mathrm{n}\}$.

Step 3. Now, to analyze the sensitivity in upper bound interval transportation problem, after obtaining the optimal solution. Let $p_t = \max.\{p_1, p_2,...., p_m\}$ and $q_k = \max.\{q_1, q_2,...., q_n\}$. In the allotment table of the upper bound interval transportation problem, we replace p_i by $p_i + \alpha_i$, i=1,2,..., m; $i \neq t$ and p_t by $p_t - \sum_{\substack{i=i \\ i \neq t}}^{m} \alpha_i$ and also, we replace q_j by $q_j + \lambda_j$, i=1,2,...,n; $j \neq k$ and q_k by $q_k - \sum_{\substack{j=i \\ j \neq k}}^{n} \lambda_j$.

Step 4. Compute the minimum and maximum values of α_i and λ_j ,for all i and j using the allotment conditions of zero point method to maintain the current optimal basis. Then, find the ranges of supply and demand values in the upper transportation problem.

Step 5. Apply the Step 2. to the Step 4. to the lower bound interval transportation problem and compute the ranges of supply and demand values in the lower bound interval transportation problem to maintain the current optimal basis.

Step 6. From the Step 4. and the Step 5., we obtain the interval ranges of the supply and demand interval in the interval transportation problem to maintain the current optimal basis.

The upper-lower method is illustrated with help of the following numerical example.

Example 4.1. Consider the fully interval transportation problem:

	D_1	D_2	D_3	Supply
S_1	[2,6]	[3,5]	[2,4]	[40,60]
S_2	[1,3]	[3,6]	[2,4]	[25,35]
Demand	[20,30]	[15,25]	[30,40]	[65,95]

Now, the lower bound and upper bound transportation problems of the given interval transportation problem are given below:

	D_1^L	D_2^L	D_3^L	Supply
S_1^L	2	3	2	40
S_2^L	1	3	2	25
Demand	20	15	30	**65**

and

	D_1^U	D_2^U	D_3^U	Supply
S_1^U	6	5	4	60
S_2^U	3	6	4	35
Demand	30	25	40	**95**

where $S_i = [S_i^L, S_i^U]$, i =1,2 and $D_i = [D_i^L, D_i^U]$, j =1,2,3.

Now, using the zero point method the optimal solution to the upper bound transportation problem is

	D_1^U	D_2^U	D_3^U	Supply
S_1^U	6	5 25	4 35	60
S_2^U	3 30	6	4 5	35
Demand	30	25	40	**95**

Now, using the zero point method, the optimal solution to the lower bound transportation problem is given below:

	D_1^L	D_2^L	D_3^L	Supply
S_1^L	2	3 15	2 25	40
S_2^L	1 20	3	2 5	25
Demand	20	15	30	**65**

The optimal solution of the fully interval transportation problem is

	D_1	D_2	D_3	Supply
S_1	[2,6]	[3,5] [15,25]	[2,4] [25,35]	[40,60]
S_2	[1,3] [20,30]	[3,6]	[2,4] [5,5]	[25,35]
Demand	[20,30]	[15,25]	[30,40]	[65,95]

Now, we study the sensitivity of the supply and demand parameters in the upper bound transportation problem:

Now, the allotment table for the upper bound transportation problem is given below:

	D_1^U	D_2^U	D_3^U	Supply
S_1^U	3	0	0	60
S_2^U	0	1	0	35
Demand	30	25	40	**95**

Now, using the Step 3. of the upper-lower method, we have the following:

	D_1^U	D_2^U	D_3^U	Supply
S_1^U	3	0	0	$60-\alpha_1$
S_2^U	0	1	0	$35+\alpha_1$
Demand	$30+\lambda_1$	$25+\lambda_2$	$40-\lambda_1-\lambda_2$	**95**

Now, using the allotment conditions of the zero point method related to supply, we have

$$60-\alpha_1 < 65-\lambda_1 \text{ and } 35+\alpha_1 \le 70-\lambda_2 \text{ (or)}$$
$$60-\alpha_1 \le 65-\lambda_1 \text{ and } 35+\alpha_1 < 70-\lambda_2 .$$

Now, using the allotment conditions of the zero point method related to demand, we have

$$30+\lambda_1 < 35+\alpha_1,\ 25+\lambda_2 < 60-\alpha_1 \text{ and } 40-\lambda_1-\lambda_2 \le 95 \text{ (or)}$$
$$30+\lambda_1 < 35+\alpha_1,\ 25+\lambda_2 \le 60-\alpha_1 \text{ and } 40-\lambda_1-\lambda_2 < 95 \text{ (or)}$$
$$30+\lambda_1 \le 35+\alpha_1,\ 25+\lambda_2 < 60-\alpha_1 \text{ and } 40-\lambda_1-\lambda_2 \le 95 .$$

Now, using the allotment conditions of the zero point method related to both supply and demand, we have the following:

$$\lambda_1-\alpha_1 \le 5$$
$$\lambda_2+\alpha_1 \le 35 \qquad \text{and}$$
$$-55 \le \lambda_1+\lambda_2 \le 40 . \qquad (4)$$

Now, since availability at each supply point and the requirement at each demand point are non-negative, we have the following:

$$-35 \le \alpha_1 \le 60 \qquad (5)$$
$$-30 \le \lambda_1 \qquad (6)$$
$$-25 \le \lambda_2 \qquad (7)$$
$$\lambda_1+\lambda_2 \le 40 \quad . \qquad (8)$$

Now, from equations (4) ,(6), (7) and (8), we have

$$-30 \le \lambda_1 \le 65 \qquad (9)$$
$$\text{and} \quad -25 \le \lambda_2 \le 70 . \qquad (10)$$

Now, from (5), we have $0 \le S_1^U \le 95$ and $0 \le S_2^U \le 95$

Now, from (9) and (10), we have $0 \le D_1^U \le 70$, $0 \le D_2^U \le 65$ and $0 \le D_3^U \le 95$.

Now, the allotment table for the lower bound transportation problem:

	D_1^L	D_2^L	D_3^L	Supply
S_1^L	1	0	0	40
S_2^L	0	0	0	25
Demand	20	15	30	**65**

Now, using the Step 3. of the upper-lower method, we have the following:

	D_1^L	D_2^L	D_3^L	Supply
S_1^L	1	0	0	$40-\alpha_1$
S_2^L	0	0	0	$25+\alpha_1$
Demand	$20+\lambda_1$	$15+\lambda_2$	$30-\lambda_1-\lambda_2$	**65**

Now, using the allotment conditions of the zero point method and non-negativity conditions of the availability at each supply point and the requirement at each demand point, we have the following:

$$-25 \le \alpha_1 \le 40,\ -20 \le \lambda_1 \le 30,\ -15 \le \lambda_2 \le 30 \text{ and } -35 \le \lambda_1 + \lambda_2 \le 30.$$

This implies that $0 \le S_1^L \le 65$ and $0 \le S_2^L \le 65$, $0 \le D_1^L \le 50$, $0 \le D_2^L \le 45$ and $0 \le D_3^L \le 65$.

Now, the ranges of all supply and demand in the given interval transportation problem such that its optimal basis is invariant, are given in the following table:

	Minimum limit	Original value	Maximum limit
Supply 1, S_1	[0, 0]	[40, 60]	[65, 95]
Supply 2, S_2	[0, 0]	[25, 35]	[65, 95]
Demand 1, D_1	[0, 0]	[20, 30]	[50, 70]
Demand 2, D_2	[0, 0]	[15, 25]	[45, 65]
Demand 3, D_3	[0, 0]	[30, 40]	[65, 95]

5. Fully Fuzzy Transportation Problem

Consider the following fuzzy integer transportation problem (FFITP) where

(FFITP) Minimize $\tilde{z} \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\tilde{c}_{ij} \otimes \tilde{x}_{ij}\right)$

subject to

$$\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i \,,\; i = 1,2,...,m$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j \,,\; j = 1,2,...,n$$

$$\tilde{x}_{ij} \geq \tilde{0}\,,\; i = 1,2,...,m \text{ and } j = 1,2,...,n \text{ and are integers}$$

where m = the number of supply points ; n = the number of demand points ; $\tilde{x}_{ij}$ is the uncertain number of units shipped from supply point i to demand point j ; $\tilde{c}_{ij}$ is the uncertain cost of shipping one unit from supply point i to the demand point j which is non-negative ; $\tilde{a}_i$ is the uncertain supply at supply point i and $\tilde{b}_j$ is the uncertain demand at demand point j.

A trapezoidal fuzzy number (a,b,c,d) can be represented as an interval number form as follows.

$$(a,b,c,d) = [a + (b-a)\alpha, d - (d-c)\alpha] \;;\; 0 \leq \alpha \leq 1. \tag{11}$$

Using the relation (11), we can convert the given fuzzy transportation problem into an interval transportation problem. Using the upper-lower method, we obtain the ranges of all supply and demand in the interval transportation problem such that its optimal basis is invariant. Then, again using the relation (11), we can find the ranges of all supply and demand in the given fuzzy transportation problem.

The solution procedure is illustrated with the following example

Example 5.1: Consider the following fully fuzzy transportation problem

	$\tilde{D}_1$	$\tilde{D}_2$	$\tilde{D}_3$	Supply
$\tilde{S}_1$	(2,3,5,6)	(3,4,4,5)	(2,3,3,4)	(40,45,55,60)
$\tilde{S}_2$	(1,2,2,3)	(3,4,5,6)	(2,3,3,4)	(25,30,30,35)
Demand	(20,23,27,30)	(15,18,22,25)	(30,34,36,40)	

Now, the fully interval transportation problem corresponding to the above problem is given below:

	D_1	D_2	D_3	Supply
S_1	[2+α ,6-α]	[3+α ,5-α]	[2+α ,4-α]	[40+5α ,60-5α]
S_2	[1+α ,3-α]	[3+α ,6-α]	[2+α ,4-α]	[25+5α , 35-5α]
Demand	[20+3α ,30-3α]	[15+3α ,25-3α]	[30+4α ,40-4α]	

Now using the upper-lower method, the sensitivity analysis of all supply and demand in the given interval transportation problem such that its optimal basis is invariant, is given in the following table.

	Minimum limit	Original value	Maximum limit
Supply 1, S_1	[0,0]	[40+5α ,60-5α]	[65+10α ,95-10α]
Supply 2, S_2	[0,0]	[25+5α , 35-5α]	[65+10α ,95-10α]

Demand 1, D_1	[0,0]	[20+3α ,30-3α]	[50+7α ,70-7α]
Demand 2, D_2	[0,0]	[15+3α ,25-3α]	[45+7α ,65-7α]
Demand 3, D_3	[0,0]	[30+4α ,40-4α]	[65+10α ,95-10α]

Now, using the relation (11), the ranges of fuzzy supply and demand of the given fuzzy transportation problem such that its optimal basis is invariant, are given in the following table:

	Minimum limit	Original value	Maximum limit
Supply 1, $\tilde{S}_1$	(0,0,0,0)	(40,45,55,60)	(65,75,85,95)
Supply 2, $\tilde{S}_2$	(0,0,0,0)	(25,30,30,35)	(65,75,85,95)
Demand 1, $\tilde{D}_1$	(0,0,0,0)	(20,23,27,30)	(50,57,63,70)
Demand 2, $\tilde{D}_2$	(0,0,0,0)	(15,18,22,25)	(45,52,58,65)
Demand 3, $\tilde{D}_3$	(0,0,0,0)	(30,34,36,40)	(65,75,85,95)

6. Conclusion

We discuss the sensitivity analysis of supply and demand in the interval transportation problem in this paper. We propose a method namely, upper-lower method for finding a critical region of the supply and demand parameters at which any change inside the ranges of the region does not affect the optimal basis, while, any change outside their ranges will affect the optimal basis. In general, an information of sensitivity analysis in a transportation problem is usually more important than the optimal solution itself. The proposed method is extended to fuzzy transportation problems. The sensitivity analysis of supply and demand parameters in an interval transportation problem by the proposed method can help the decision makers to know in what range of variation of sources in the market they can keep the installed production lines active, and only production's levels would change when they are handling distribution problems having imprecise parameters.

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