Synthesis of *One* Degree-Of-Freedom 6-Bar Linkages from *Three* Degree-Of-Freedom Open 4-Bar Chain Using Structural Code Technique

H.Eleashy* , M.Samy Elgayyar, M.N. Shabara*****
*Mechanical Engineering Dept., Future University In Egypt,Cairo, Egypt
**Production& Mechanical Design Dept., Mansoura university, Mansoura, Egypt
***Production& Mechanical Design Dept., Mansoura university, Mansoura, Egypt

ABSTRACT
This paper presents a systematic methodology to convert planar 3 degree-of-freedom open 4-bar chain into planar 1 degree-of-freedom 6-bar linkages. This objective is achieved by adding 2RR chains to the original mechanism. Hence, these additional chains mechanically constrain the relative movement of the end-effector to reach a required set of task positions. Graphical enumeration technique is applied for structural synthesis of all possible 6-bar linkages. Structural code technique is introduced as a new method for structural isomorphism. The proposed methodology produces 7 different forms of 6-bar linkages.

***Keywords**- structural synthesis, six-bar linkages, 3R chain, graphical enumeration.*

1. Introduction
Planar four-bar mechanism is the most practical and widely implemented mechanism in different mechanical systems applications. An open 4-bar chain can be considered a planar robot that formed by a serial chain constructed from three revolute joints with 3 degree-of-freedom. Additional 2 RR chains convert the original system to a single degree of freedom system. Also these additional chains constrain the relative motion of the end-effector to achieve the required task positions. This work is inspired by Krovi et al. (2002) [4], who derived synthesis equations for planar nR planar serial chains in which the n joints are constrained by a cable drive. They obtained a "single degree-of-freedom coupled serial chain" that they use to design an assistive device. Kinematic analysis of planar mechanisms are presented by Bottema , Roth 1990 [1]. Tsai (2001) [2] produce a systematic methodology for the creation and classification of mechanisms using analytical approach. Some of the functional requirements of a desired mechanism can be transformed into structural characteristics that can be employed for systematic enumeration of mechanisms. McCarthy et al.(2006) [9] discuss the problem of synthesis of a mechanically constrained 3R chain designs two RR chains to reach five task positions. The 3R chain has been sized by McCarthy et al.(2008) [12] to reach the five task positions, then two RR chains may be attached to obtain seven different forms of a six-bar linkage. This procedure results in as many as 63 candidate designs. The isomorphism of mechanism kinematics chain has been identified by comparing the eigenvalues and corresponding eigenvectors of adjacent matrices by Yuxin Wang (2002) [3] . Zhen Huang (2009) [13] discuss Isomorphism identification of graphs by finding a unique representation of graphs to solve the problem of isomorphism in mechanism synthesis process.

In the following sections graphical representation and enumeration are presented to obtain all ways that two RR chains may be added to a 3R chain. The proposed structural code technique is presented to detect isomorphic graphs. Finally, reverse transformation process is applied to get the corresponding linkage diagrams for enumeration results.

2. Graphical representation for 3R chain
An illustration of the ways that a 3R serial chain can be constrained to obtain a one degree-of- freedom system using graphical techniques. Originally the 3R chain is represented graphically by identifying each link as a vertex, and each joint as an edge as shown in Figure 1. The linkage graph is constructed by identifying each link as a vertex, and each joint as an edge.

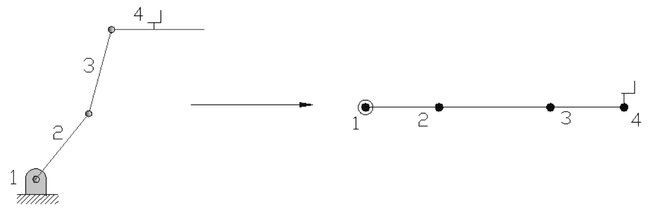

Figure 1. A schematic of a planar 3R robot and its corresponding graphical representation

3. Synthesis of Mechanically Constrained 3R Chains
The planar 3R robot consists of four bars. The introduction of an RR chain adds a vertex and two edges to the graph. Therefore, two vertices and four edges will be added to the initial graph that represents the 3R chain. Therefore, the appropriate attachment of two RR chains results in a planar mechanically constrained mechanism having six links, seven revolute joints and one degree of freedom. Synthesis of a mechanically constrained 3R chain proceeds in two main steps. The first step is to enumerate all possible and non isomorphic 6-link graph representations. Then these graph representations are reverse transformed into linkage diagrams of planar 6-bar mechanisms.

4. Graphical Enumeration for 6-bar linkages

The two additional links may be dependent in their connection if there is a revolute connection will connect them. They may be independent if there is no revolute joint will connect them as shown in Figure 2.

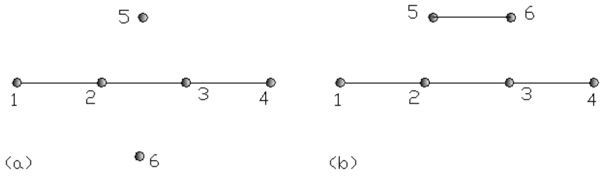

Figure 2. a- independent link connection.
b- dependent link connection

4.1. Enumeration of independent link connection

We can summarize enumeration procedures in the following steps:

Step 1. The two additional links, 5 and 6, are represented.

Step 2. Each additional link can be connected to the original four links by 4 revolute joints in such a way that both of them should be connected to only one original link by one revolute joint. In other words each additional link should have at least one revolute joint, this will enumerate 16 available candidates shown in Figure 3.

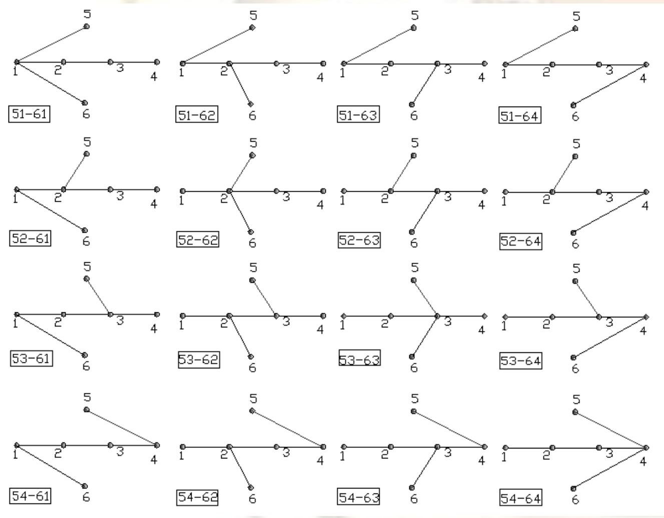

Figure 3. Available candidates after adding the first two revolute joints for independent link connections

Step 3. Initial structural code is deduced for each candidate. This code consists of primary and secondary codes. Primary code represents the additional link "5" and one original link that have a revolute joint with link "5". Secondary code represents the additional link "6" and one original link that have a revolute joint with link "6". Therefore, the second candidate, for example, will have a primary structural code of "51" and secondary code of "62". Each candidate is labeled by its initial structural code as shown in Figure 3.

Step 4. Another two revolute joints are added for each candidate such that each additional link has one revolute joint. This will produce 15 graphs for each candidate. Each of additional links 5,6 must has two revolute joints with the original links 1,2,3 and 4. This will exclude 6 rejected graphs in which one of the additional links has three revolute joints with the original links. This produces nine available constructions.

Step 5. Therefore, there will be 144 available graphs from this enumeration process. Some of these results are shown in Figure 4.

Step 6. Structural code should be extended to represent the second RR chain. The second graph has a primary code of "512" such that link 5 has revolute joints with links 1 and 2. The secondary code of the same graph is "613" such that link 6 has revolute joints with links 1 and 3. All enumerated graphs have their corresponding structural code as shown in Figure 4.

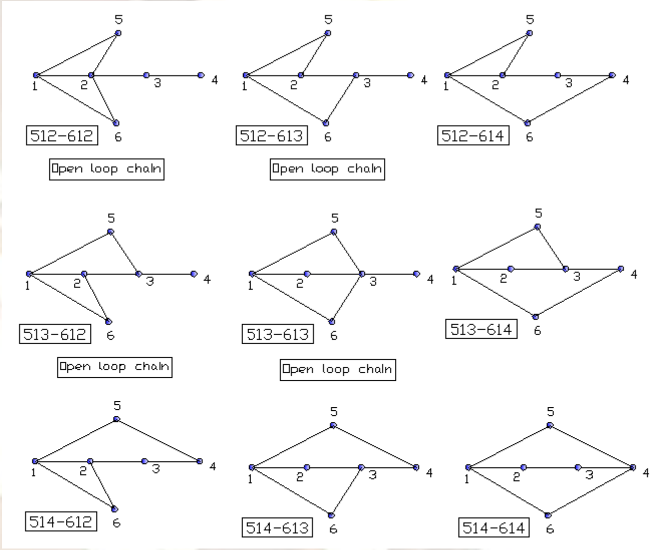

Figure 4. Some results of Enumeration of independent link connection for the first candidate "51-61"

4.1.1 Detect open loop chains using structural code

Link 4 that represents the end effector should have at least 2 revolute joints with other links unless this mechanism is considered an open loop mechanism and will not be mechanically constrained. Structural code is applied to detect all open loop mechanisms such that if both primary and secondary structural codes of any graph don't contain link "4". These graphs are detected in Figure 4 and then should be excluded from enumeration process.

4.1.2 Isomorphism using structural code

Isomorphic graphs are detected using structural code technique. Two graphs are isomorphic if the first two digits of both primary and secondary codes are identical or reversed. For example, the third graph in Figure 4 has a structural code of "512-614" and the seventh graph has a structural code of "514-612". The first two digits of their structural codes, "12-14", are the same but reversed. Hence, the two graphs are said to be isomorphic. Each group of isomorphic graphs will be represented only once by only one graph in the enumeration results. As a result, there will be 15 non-isomorphic graphs shown in Figure 5.

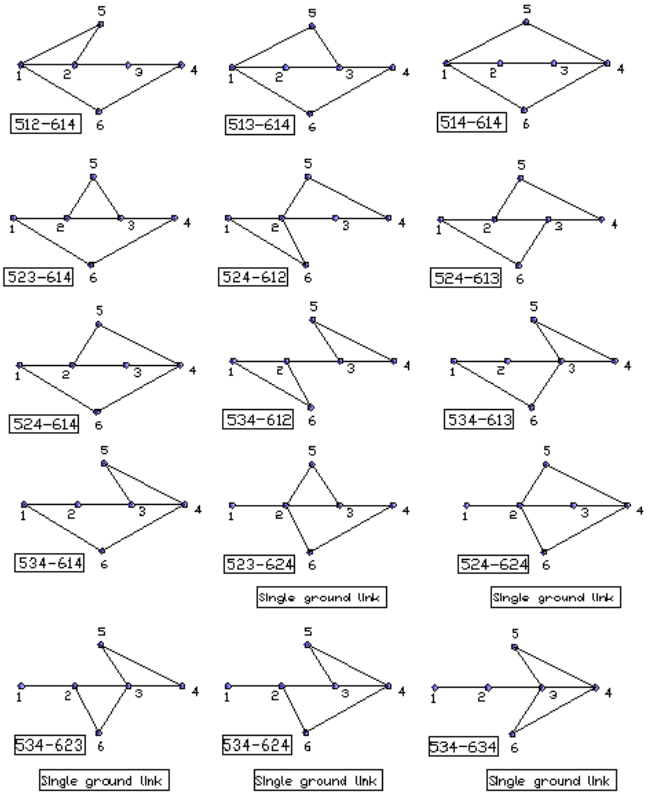

Figure 5. Non- isomorphic graphs for independent link connection

4.1.3 Detect graphs with single ground link

After adding the 2 RR chains, at least one of the two additional links 5 or 6 must has one revolute joint with the ground link unless this produces an open loop chain which cannot be mechanically constrained. Graphs with single ground link are detected using the proposed structural code. Any graph will have single ground link if its structural code does not contain link "1". These graphs are detected as shown in Figure. 5 and then eliminated from enumeration process. The final results of enumeration process are ten graphs, shown in Figure 6, having no single ground link, no isomorphic graphs and no open loop chains.

4.1.4 Reverse Transformation

As referred before in graphical representation links are denoted as vertices and revolute joints are denoted as thin edges. Each graph from the ten final graphs will be transformed back to its original mechanism or its linkage graph representation. Number of common edges at the same vertex indicates the type of link connection. Figure 7 illustrates the first graph representation and its corresponding mechanism. For example vertex 2 has three thin edges with vertices 1, 4, 5. So that, link 2 must be ternary link with three revolute joints with links 1,4,5. Also vertex 5 has only two thin edges with vertices 1, 2. So that link 5 must be binary link with two revolute joints with links 1,2.

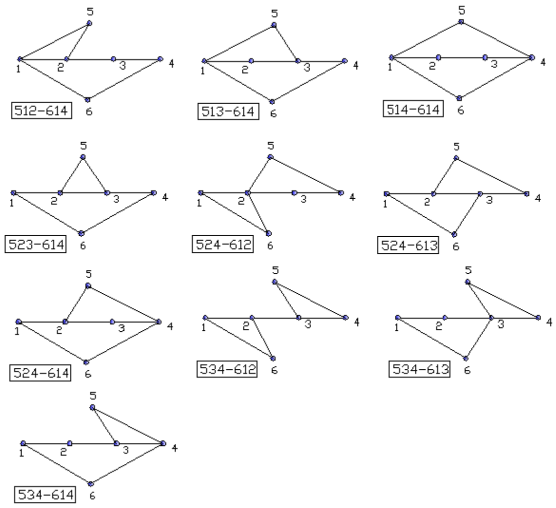

Figure 6. Final enumeration results for independent link connection

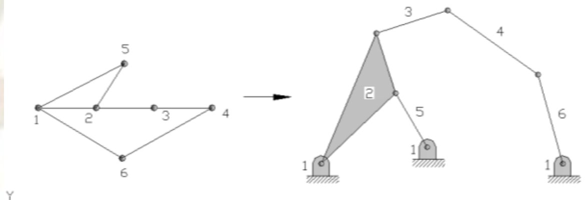

Figure 7 Reverse transformation process

The same procedures are performed to have all results of enumeration process. Figure 8 shows the corresponding ten mechanisms or linkage diagrams resulting from transformation process.

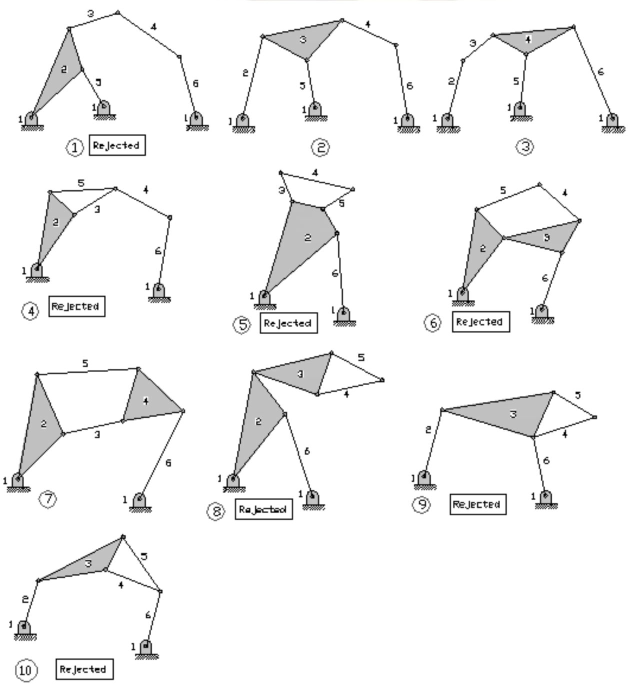

Figure 8. Corresponding linkage diagrams for final enumeration results of independent link connections

To ensure relative movement of links for each mechanism, any locked chains should be detected and its corresponding mechanism will be rejected. Locked chains can be detected graphically if there are three revolute joints connecting three consecutive links. Applying these rules to the resulting ten mechanisms, seven mechanisms will be excluded and finally we have three available 6-bar mechanisms, Stephenson IIIa, Stephenson IIIb and Stephenson IIb. Figure 9 illustrates these results.

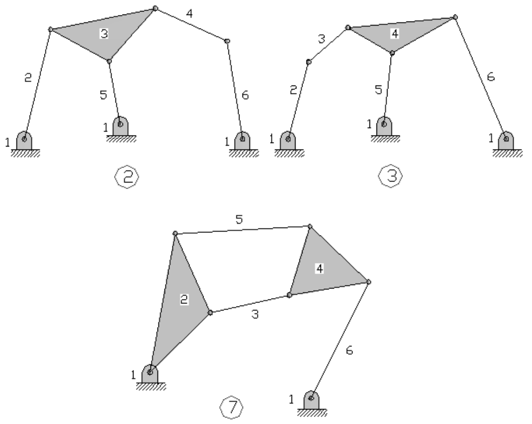

Figure. 9. Available 6-bar mechanisms for independent link connection

4.2 Enumeration for dependent link connection

We can summarize enumeration procedures in the following steps:

Step1. The two additional links, 5 and 6, are represented with their revolute connection.

Step2. Add the second revolute joint only by all 8 deferent ways as shown in Figure 10.

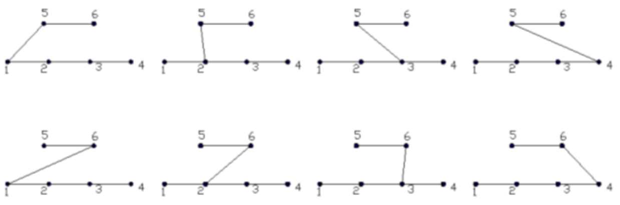

Figure 10. Available candidates after adding the first two revolute joints for dependent link connections

Step3. The last two revolute joints are added to each structure such that they connect original links with the link which has no joints with original links. This will produce six available candidates for each structure.

Step4. Therefore, there will be 48 available graphs from this enumeration process. Some of these results are shown in Figure 11**.**

Step5. *Structural codes* are presented based on the connections of links 5 and 6 with original links. The first graph for example has a structural code of "51-612".

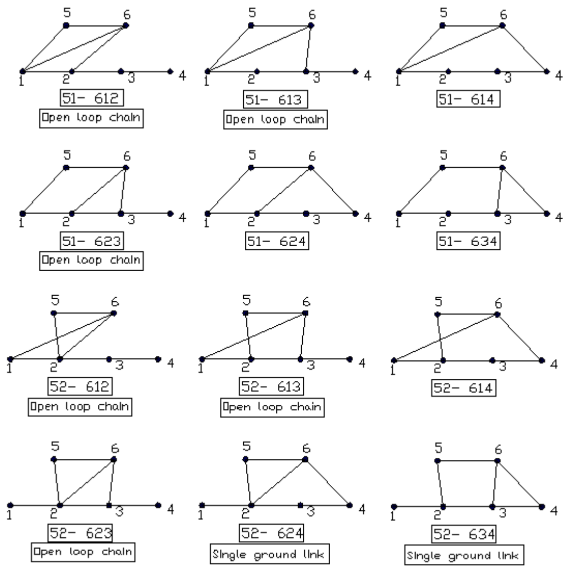

Figure 11. Some results of Enumeration of dependent link connection

4.2.1 Detect open loop chains using structural code

Using the proposed structural code**,** all open loop mechanisms are detected, as shown in Figure. 11, and then excluded from enumeration process.

4.2.2 Detect graphs with single ground link using structural code

All graphs with single ground link are detected using the proposed structural code technique as shown in Figure 11. These graphs are then eliminated from enumeration process.

4.2.3 Isomorphism using structural code

Using the structural code technique, eight non isomorphic graphs can be detected as shown in Figure 12.

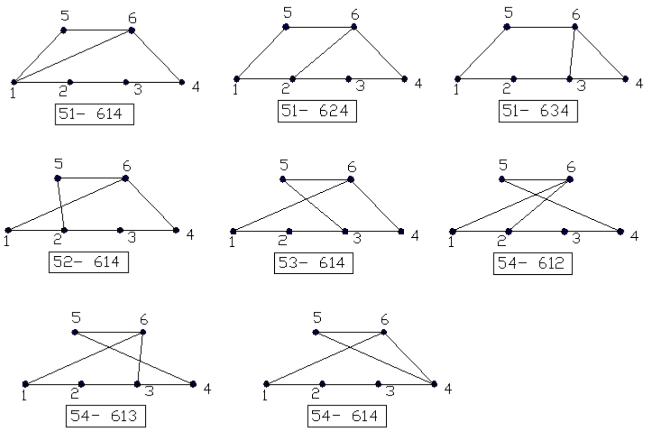

Figure 12. Final enumeration results for dependent link connection

4.2.4 Reverse Transformation

Each graph from the eight final graphs will be transformed back to its original schematic mechanism representation.

The corresponding linkage mechanisms for these eight structures are shown in Figure 13.

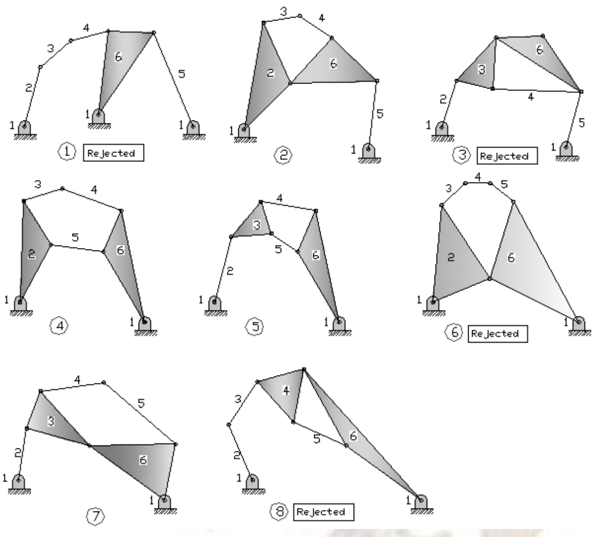

Figure13. Corresponding linkage graphs for final enumeration results for dependent link connection

4.2.5 Check for kinematic characteristics

Applying the same rules of detecting locked chains, four mechanisms will be rejected and excluded from enumeration process. Finally we have four available 6-bar mechanisms: Watt Ib, Stephenson I, Stephenson IIa and Watt Ia as shown in Figure 14.

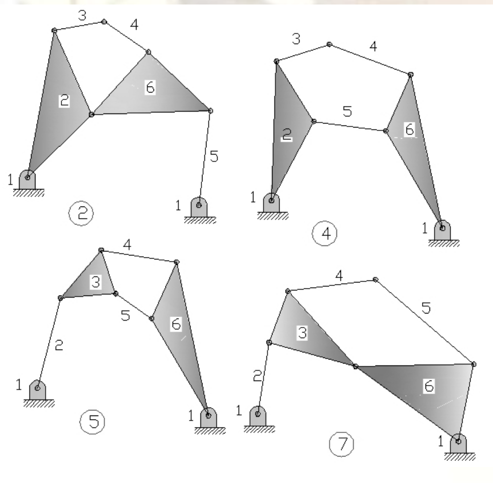

Figure 14. Available 6-bar mechanisms for dependent link connection

Results

An open 4-bar mechanism with 3 degree-of-freedom is converted to a single degree-of-freedom 6-bar mechanism by adding 2 RR chains. These 2 RR chains mechanically constrain the movement of end-effector link. The proposed structural code technique is applied to detect open graphs, graphs with single ground links and isomorphic graphs. Structural synthesis results in *three* 6- bar mechanisms from independent link connection. *Four* 6-bar mechanisms have been synthesized from dependent link connections.

Conclusion

In this paper graphical representation and enumeration are presented to synthesis 6-bar mechanisms constraining the movement of the 3R chain. Adding the 2 RR chains produces seven non-isomorphic 6-bar structures. A new structural code technique has been introduced as an easy and efficient tool in enumeration process. Systematic reverse transformation process has been presented to transform these graphs into linkage graphs.

References

[1] Bottema, O., and Roth, B., Theoretical Kinematics, North Holland Press, NY. 1990

[2] L.W. Tsai, Enumeration of Kinematic Structures According to Function, CRC Press. , 2001

[3] Zongyu Chang , Ce Zhang, Yuhu Yang, Yuxin Wang , A new method to mechanism kinematic chain isomorphism identification, Mechanism and Machine Theory 37 ,2002, 411–417

[4] Krovi, V., Ananthasuresh, G. K., and Kumar, V., “Kinematic and Kinetostatic Synthesis of Planar Coupled Serial Chain Mechanisms,” ASME Journal of Mechanical Design, 2002, 124(2):301-312.

[5] Venkat Krovi,G. K. Ananthasuresh ,Vijay Kumar, Kinematic and Kinetostatic Synthesis of Planar Coupled Serial Chain Mechanisms, Journal of Mechanical Design ASME JUNE, 2002, Vol. 124

[6] Perez, A., and McCarthy, J. M., 2005, “Clifford Algebra Exponentials and Planar Linkage Synthesis Equations” ASME Journal of Mechanical Design, 127(5):931-940, September.

[7] P.S. Shiakolas, D. Koladiya, J. Kebrle, On the optimum synthesis of six-bar linkages using differential evolution and the geometric centroid of precision positions technique, Mechanism and Machine Theory 40 (3) (2005) 319–335.

[8] Yi Lu, Tatu Leinonen, Type synthesis of unified planar–spatial mechanisms by systematic linkage and topology matrix-graph technique, Mechanism and Machine Theory 40 (2005) 1145–1163

[9] Soh, Perez, and McCarthy, The Kinematic Synthesis of Mechanically Constrained Planar 3R Chains, European Conference on Mechanism Science, Obergurgl (Austria), February 21–26 2006

[10] H.F. Ding Z. Huang, A Unique Matrix Representation for the Kinematic Chain , 12th IFToMM World Congress, Besançon (France), June18-21, 2007

[11] Huafeng Ding , Zhen Huang,A new theory for the topological structure analysis of kinematic chains and its applications, Mechanism and Machine Theory 42 (2007) 1264–1279

[12] Gim Song Soh , J. Michael McCarthy, The synthesis of six-bar linkages as constrained planar 3R chains , Mechanism and Machine Theory 43 (2008) 160–170

[13] Huafeng Ding , Zhen Huang, Isomorphism identification of graphs: Especially for the graphs of kinematic chains , Mechanism and Machine Theory 44 (2009) 122–139

[14] Abdullah F. Al-Dwairi , Fikri T. Dweiri , Omar M. Ashour,A novice-centered decision-support system for type synthesis of function-generation mechanisms , Mechanism and Machine Theory 45 (2010) 1252–1268

[15] Huafeng Ding, Jing Zhao , Zhen Huang, Unified structural synthesis of planar simple and multiple joint kinematic chains, Mechanism and Machine Theory 45 (2010) 555–568