Structural Synthesis of Mechanically Constrained Single Loop 6-Bar Chain

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ABSTRACT

In the present work, graphical enumeration technique is applied for structural synthesis of mechanically constrained planar parallel robot formed by single loop 6-bar chain or two 3R chains. Two RR chains are added to the considered system to mechanically constraint the movement of its end-effector. Structural code has been proposed as a new systematic methodology to detect isomorphic graphs. Therefore, 32 non-isomorphic graphs are obtained from this enumeration process. All these graphs represent one-degree-of freedom eight-bar mechanisms. Reverse transformation process is presented to obtain the corresponding linkage diagrams for all results.

Keywords- *structural synthesis, eight-bar linkages, parallel robot, graphical enumeration*

1. Introduction

For many modern applications, a planar parallel robot has been used as an efficient and accurate system. The parallel robot formed by an end-effector supported by two 3R chains defines a single loop six-bar chain. This system has 3 degree-of-freedom. The objective of this work is to mechanically constrain the relative movement of the joints so that the end-effector reaches a specified set of task positions. Two RR chains will be added to mechanically constrain the considered system. This will produce eight bar linkages with one degree of freedom. Tsai (2001) [1] produce that there are sixteen topologies for one degree-of-freedom eight bars linkages. McCarthy et al.(2006) [2] add two RR chains to a planar robot formed by 3R chain. Therefore the system has been mechanically constrained. Also the problem reaching five task positions by this system has been discussed. Elgayyar et al.(2006) [3] used structural code technique to detect isomorphic graphs in the enumeration process of epicyclic gear trains. The 3R chain has been sized by McCarthy et al.(2008) [4] to reach the five task positions, then two RR chains may be attached to obtain seven different forms of a six-bar linkage. This procedure results in as many as 63 candidate designs. Zhen Huang (2009) [5] discuss Isomorphism identification of graphs by finding a unique representation of graphs to solve the problem of isomorphism in mechanism synthesis process.

Krovi et al. (2002) [6] derived synthesis equations for planar nR planar serial chains in which the n joints are constrained by a cable drive.

In the following sections we will introduce systematic procedures to enumerate all possible eight bar linkages from an original single six-bar loop. Then kinematic characteristics of planar mechanisms will be applied. Structural code technique is presented to detect isomorphic graphs. Finally reverse transformation process is applied for all enumeration results to get all eight bar linkages.

2. Graphical representation for planar parallel robot

One 3R chain is represented graphically by identifying each link as a vertex and each joint as an edge. The linkage diagram is constructed by identifying each link as a vertex, and each joint as an edge. If two 3R chains are combined to form a planar parallel robot, this will produce single loop six bar chain which has two fixed links. Linkage diagram of this system and its corresponding graph representation are shown if Figure 1.

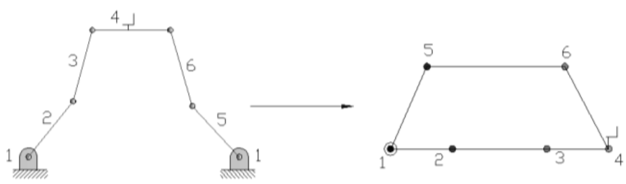

Figure 1. Linkage diagram of a planar parallel robot and its corresponding Graphical representation

3. Synthesis of Mechanically Constrained 8-bar linkages

The planar parallel robot consists of single loop six bar chain which has two fixed links including its base. This system can be mechanically constrained by adding two links and four revolute joints to maintain its degree of freedom. Therefore, the appropriate attachment of two RR chains results in a planar eight-bar linkage with ten revolute joints forming a one degree-of-freedom system. The two RR chains must be added in such a way that the system remains with only two fixed joints and also it should contains no locked chains. The corresponding graph should be modified by adding two vertices and four edges to the initial graph that represents the planar parallel robot.

4. Graphical Enumeration for 8-bar chains

Considering all ways in which two vertices and four revolute joints may be added. Figure 2 (a) shows a graph representation for original parallel robot after adding two links 7 and 8. One of the four revolute joints may connect links 7 and 8. The arrangement, in this case, is called *dependent link connection*. If there is no revolute joint will connect links 7 and 8, the arrangement is called *independent link connection* as shown in Figure 2(b).

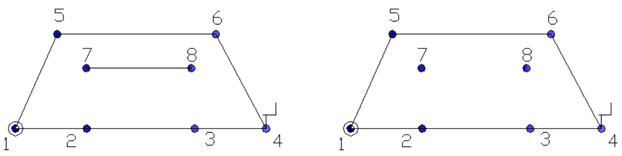

Figure 2. (a) Dependent link connection.
(b) Independent link connection

4.1. Enumeration of independent link connection using structural code

Links 7 and 8 will be connected to the original links 1, 2,3,4,5 and 6 with four revolute joints. To avoid open loop chain, each of links 7 and 8 should have two revolute joints with the original links. Enumeration procedures can be summarized in the following steps:

1. The first two revolute joints connect link 7 with the original links 1,2,3,4,5 and 6. There are six possible ways to connect link 7 with the original links, therefore we have $(_{2}^{6})$ =15 arrangement as shown in Figure 3.
2. For each arrangement resulting from the first step, the second two revolute joints will be added. They will connect link 8 with the original links. Hence, we have $(_{2}^{6})$ =15 structures resulting from each arrangement. Figure 4 illustrates the fifteen structures for the seventh arrangement "724". This will enumerate 300 available structures for all arrangements.

4.1.1 Structural code

Each arrangement in Figure 3 is labeled by its own primary structural code. This code consists of three digits; the third digit refers to link “7" and the first two digits refers to the original links that are connected to link "7". For the first arrangement link 7 is connected to links 1 and 2, so that its structural code is”712” and so on. Each structure in Figure 4 has its secondary structural code in addition to its primary code. Secondary structural code refers to link 8 and its connections with the original links. In the first graph as shown in Figure 4, link 8 has revolute joints with original links 1 and 2. Hence its secondary structural code is “812”.

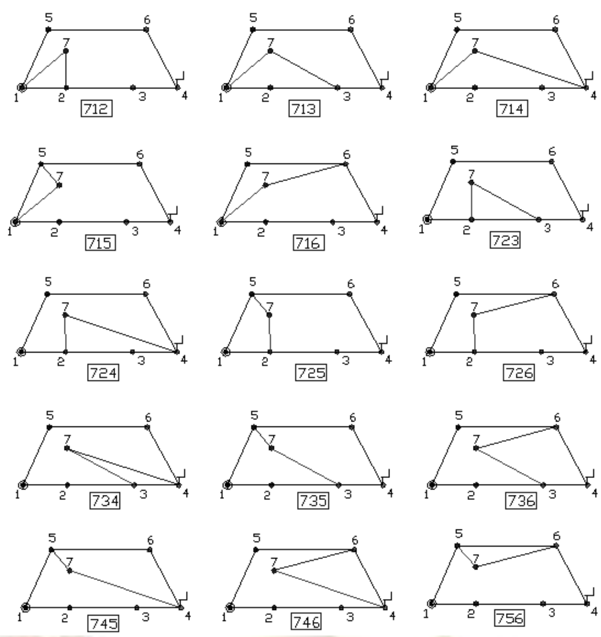

Figure 3. Fifteen arrangements for 1st step in independent link connection

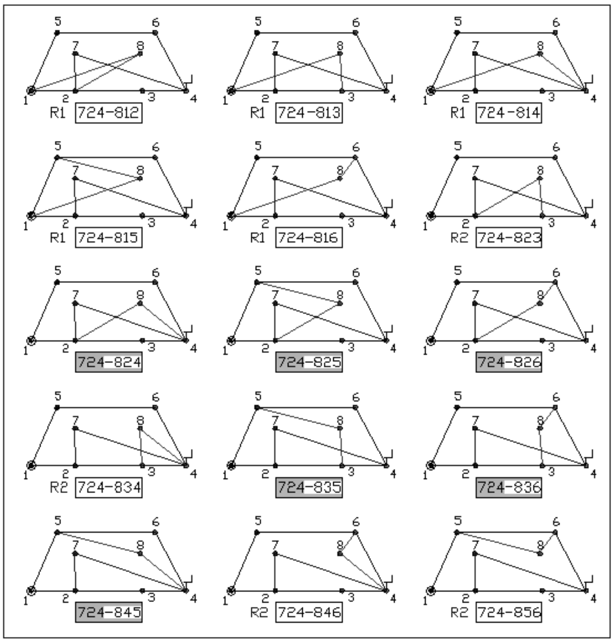

Figure 4. Enumeration results for the seventh arrangement of independent link connection

4.1.2 Detect graphs with more than two fixed pivots

Since the original planar parallel robot has only two fixed pivots, any enumerated graph having more than two connections with ground link will be detected and then rejected from enumeration results. All these graphs are labeled by "R1" as shown in Figure 4.

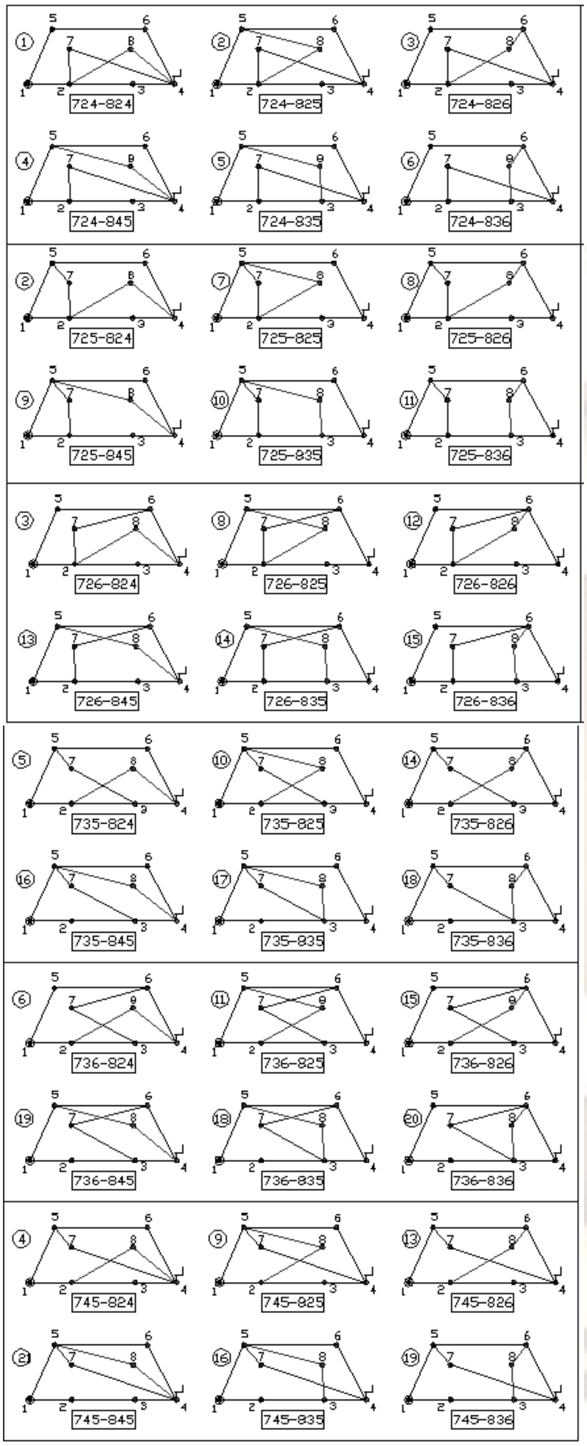

Figure 5. Enumeration results for independent link connection

4.1.3 Detect graphs with constrained two consecutive links

If one of the additional links 7 or 8 is connected with two consecutive original links, this will produce a locked chain consists of three links. All graphs containing locked chains are detected and rejected from enumeration process. These graphs are labeled by "R2" as shown in Figure 4.

The remaining 6 graphs in Figure 4, shaded on its structural code, may represent available eight bar linkages. Applying the previous detections to all of the 300 graphs resulting from enumeration process, this will produce 36 structures having only two fixed pivots and do not contain any locked chains. Figure 5 illustrates these results.

4.1.4 Isomorphism using structural code technique

In this section, the proposed structural code technique will be applied. Depending on the primary and secondary structural codes for each graph, all graphs are compared with each other. Two graphs are said to be isomorphic if the first two digits of primary and secondary structural codes, for the compared graphs, are identical or inversed. This concept may be illustrated in Figure 6.

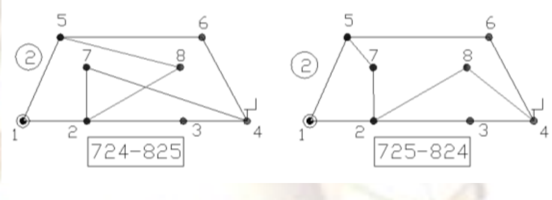

Figure 6. The concept of Isomorphic graphs for independent link connection

The two graphs shown have isomorphic group number "2". The graph on the left has a structural code of "724-825" and the other one has a structural code of "725-824". If we compare the first 2 digits of primary and secondary structural codes for both graphs, "24-25" and "25-24", we can deduce that they are inversed. Therefore they are two isomorphic graphs and they must be represented only once in the final results of enumeration process. Applying this technique, there will be 21 non isomorphic graphs that are indicated in Figure 7

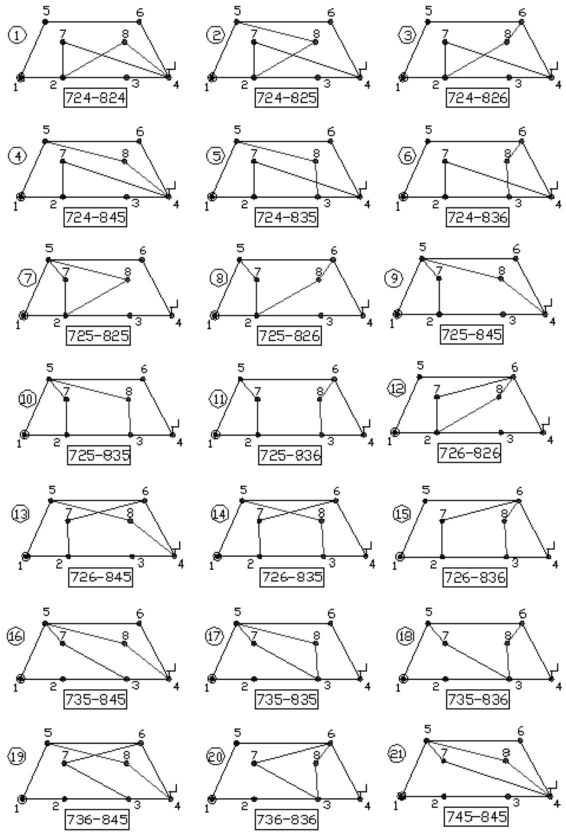

Figure 7. Non isomorphic graphs for independent link connection

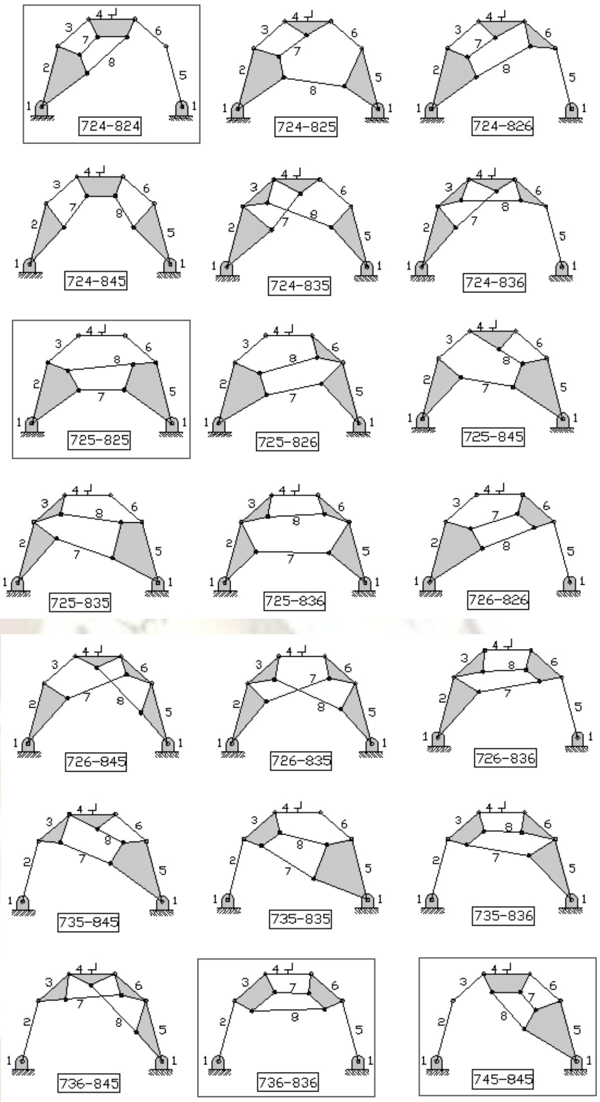

Figure (9) Corresponding linkage diagram for enumeration results of independent link connection

As referred before in graphical representation, links are denoted as vertices and revolute joints are denoted as thin edges. Each graph from the 21 final graphs will be transformed back to its original mechanism or its linkage diagram. Number of common edges at the same vertex indicates the type of link connection. Figure 8 illustrates the first graph representation and its corresponding mechanism. For example vertex 2 has 4 thin edges with vertices 1, 7, 8 and 3. So that, link 2 must be quaternary link with 4 revolute joints with links 1, 7, 8 and 3. Also vertex 8 has only 2 thin edges with vertices 2 and 4. So that link 8 must be binary link with 2 revolute joints with links 2 and 4.

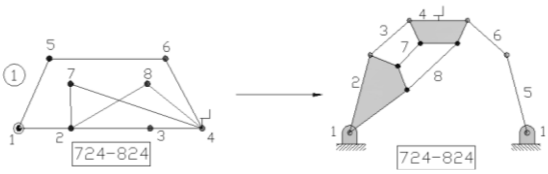

Figure 8. Reverse transformation

The same procedures are performed to have all 21 linkage diagrams. Figure 9 shows the corresponding 21 mechanisms resulting from transformation process.

associated with any loop can be estimated using the number of links, number of joints, and type of joints making up that loop. The degrees of freedom associated with any loop must be at least equal to one, to ensure that no part of a mechanism is locked. In other words, there must be a sufficient number of links and joints in each loop so that it does not form a rigid structure or locked chain. Applying this rule for the above results, we can deduce that graphs or linkages which have the structural code of "724-824", "725-825", "736-836" and "745-845" contain a locked chain. These four structures should be eliminated from enumeration results. Therefore, as final results, there are *17 available eight-bar linkage* that can constrain the movement of single loop six-bar chain by adding two independent links.

4.2 Enumeration for dependent link connection using structural code

In dependent link connection, the first revolute joint connects links 7 and 8. The remaining three joints cannot be connected with only one of links 7 or 8; otherwise we will obtain an open chain. So that, one of the additional links 7 or 8 will connect the original links by one revolute joint and the other link will connect them by two revolute joints. We can summarize enumeration procedures in the following steps:

1. The first revolute joint connects links 7 and 8.
2. The second revolute joint will connect link 7 with original links by one revolute joint. There are 6 arrangements for this connection that are shown in Figure 10. Each arrangement is labeled by its own primary structural code. This code refers to the added link "7" and the original link which is connected to link 7. For the first arrangement link 7 is connected to links 1, so that its structural code is"71" and so on.

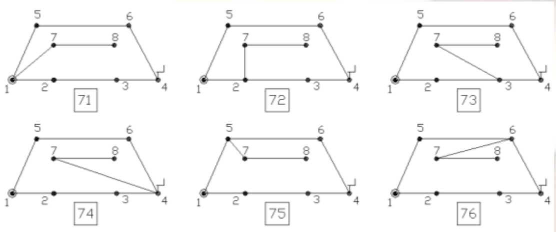

Figure 10. Available arrangements for connecting link 7 with original links in dependent link connection

3. For each arrangement, the remaining two revolute joints will connect link 8 with two of the original links. Since there are 6 ways for each joint, there are $\binom{6}{2}$ =15 available structures for each arrangement. This will produce 6*15=90 structures for all arrangements. Figure 11 illustrates the available 15 structures for the arrangement "72" in Figure 10. Each structure has its secondary structural code in addition to its primary code. Secondary structural code refers to link 8 and its connections with the original links. In the second graph shown in Figure 11, link 8 has revolute joints with original links 1 and 3. Hence its secondary structural code is "813".

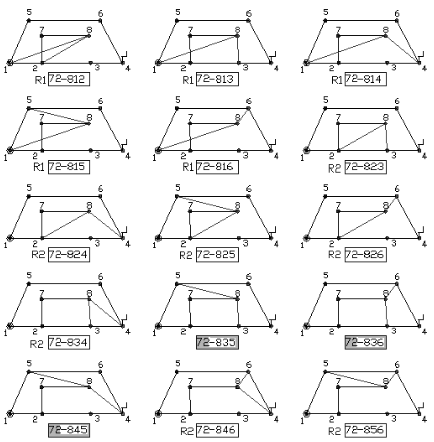

Figure 11. Fifteen available structures for arrangement "72" of dependent link

4.2.1 Detect graphs with more than two fixed pivots

To maintain the structural characteristics of the original planar parallel robot, graph having more than two connections with ground link will be detected and then rejected from enumeration results. All these graphs are labeled by "R1" as shown in Figure 11.

4.2.2 Detect graphs with constrained two consecutive links

To avoid locked chain that consists of three links, all graphs containing locked chains are detected and rejected from enumeration process. These graphs are labeled by "R2" as shown in Figure 11.

The remaining 3 graphs may represent available eight bar linkages for the second arrangement. Applying the previous detections to all of the 90 graphs resulting from enumeration process, this will produce 18 structures having only two fixed pivots and do not contain locked chains. Figure 12 illustrates these results.

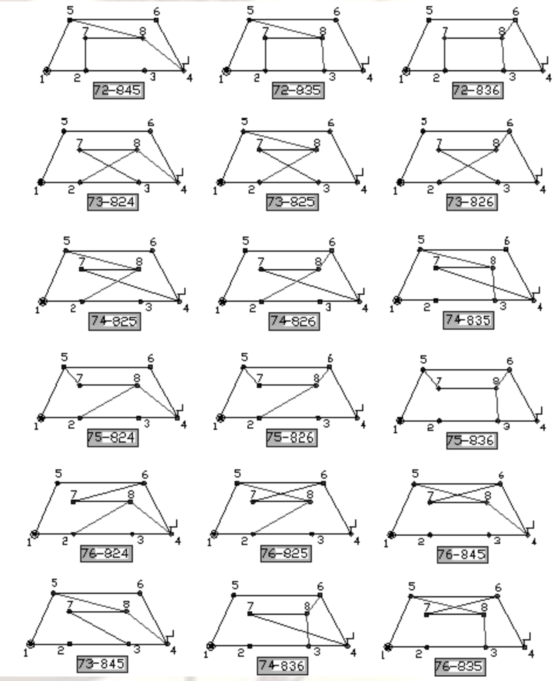

Figure 12. Enumeration results for dependent link connection

4.2.3 Reverse Transformation.

Each graph from the 18 final graphs will be transformed back to its original mechanism or its linkage diagram. Corresponding linkage diagrams resulting from transformation process are presented in Figure 13.

4.2.4 Check for kinematic characteristics.

Graphs having any locked chains are detected. We can deduce that graphs which have the structural codes of "73-824", "74-836" and "76-845" contain a rigid structure. These three structures should be eliminated from enumeration results. Therefore, as final results, there are *15 available eight-bar linkage* that can constrain the movement of single loop six bar chain by adding two dependent links.

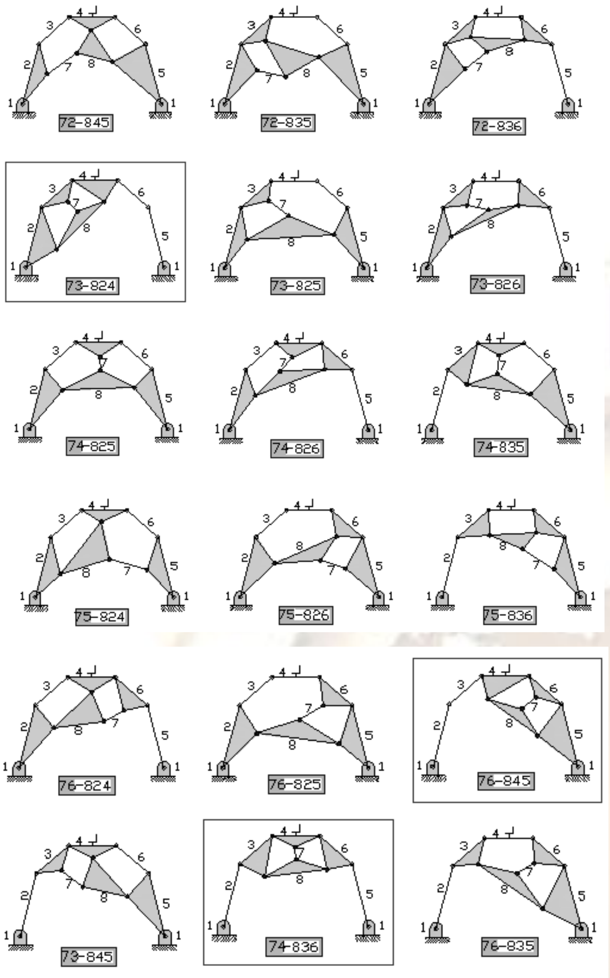

Figure 13. Corresponding linkage diagram for enumeration results of independent link connection

Results

A planar parallel robot consists of two 3R chains or single loop six-bar chain is discussed in this paper. To mechanically constraint this system, 2 RR chains have been added to the original system by all possible ways. Graphical representation and enumeration is presented as an efficient tool in structural synthesis of planar mechanisms. Therefore, the considered system is converted into planar eight-bar mechanism which has one degree of freedom and not more than two fixed pivots. A new structural code has been introduced for each enumerated graph to identify isomorphic graphs. Independent link connection results in *17* non isomorphic eight-bar mechanisms. Dependent link connection results in *15* non isomorphic eight bar mechanisms. The enumerated graphs are then transformed back into their linkage diagrams.

Conclusion

In this paper, an easy and efficient methodology is proposed to structural synthesis of eight-bar linkages to mechanically constraint the movement of planar parallel robot. A new structural code technique has been introduced to detect isomorphism. The proposed methodology produces 32 eight-bar linkages as mechanically constrained parallel robot. A systematic technique has been applied to transform the enumerated graphs to their corresponding linkage diagrams.

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