

NONLINEAR MODEL PREDICTIVE CONTROL WITH NEURAL NETWORK OPTIMIZATION FOR MECHANICAL VENTILATION OF CRITICAL CARE PATIENTS.

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ABSTRACT

Artificial breathing plays an important game in human life in the case of respiratory failure. Causes of respiratory failure will harvest instability in patient's breathing condition due to nonlinearity of gas exchange in various alveoli sac of the lung and henceforth it is necessary to pay a careful attention for several compartmental model of the lung during ventilation. This paper presents a solution for the spontaneous breathing of critical care patients under mechanical ventilation using a novel nonlinear model predictive controller enhanced by a neural network optimization. Knowledge of nonlinear dynamic system provides the way to design the mathematical view of a multi compartment lung model, which act as a plant model for the controller. Specifically the control input pressure relies on patient's physiological characteristics capturing lung resistance and compliance uncertainty of an individual compartment of the designed lung model and the various response of the respiratory system can be analyzed for determining the effectiveness of this controller. Finally the potency of controller action ties with safety constraint limit such as saturation and integral constrain into a single package and show their performance on the designed lung model for the sake smooth spontaneous breathing of the patients.

Keywords — Mechanical ventilation, multicompartment lung model, model predictive controller, nonlinear dynamic system, neural network optimization.

I. INTRODUCTION

The lungs are particularly vulnerable to acute critical illness. Respiratory failure can result not only from primary lung pathology, such as pneumonia, but also as a secondary consequence of heart failure or inflammatory illness, such as sepsis or trauma. When this occurs, it is essential to support patients while the fundamental disease process is addressed. For example, a patient with pneumonia may require mechanical ventilation while the pneumonia is being treated

with antibiotics, which will eventually effectively "cure" the disease. Since the lungs are vulnerable to critical illness and respiratory failure is common, support of patients with mechanical ventilation is very common in the intensive care unit. The goal of mechanical ventilation is to ensure adequate ventilation, which involves a magnitude of gas exchange that leads to the desired blood level of carbon dioxide (CO₂), and adequate oxygenation, which involves a blood concentration of oxygen that will ensure organ function. Achieving these goals is complicated by the fact that mechanical ventilation can actually cause acute lung injury, either by inflating the lungs to excessive volumes or by using excessive pressures to inflate the lungs. The challenge to mechanical ventilation is to produce the desired blood levels of CO₂ and oxygen without causing further acute lung injury. The earliest primary modes of ventilation can be classified, approximately, as volume-controlled or pressure-controlled [1]. In volume-controlled ventilation, the lungs are inflated (by the mechanical ventilator) to a specified volume and then allowed to passively deflate to the baseline volume.

With the increasing availability of microchip technology, it has been possible to design mechanical ventilators that have control algorithms which are more sophisticated than simple volume or pressure control. Examples are proportional assist ventilation [3], adaptive support ventilation [5], Smart Care ventilation [4], and neurally adjusted ventilation [6]. In proportional-assist ventilation, the ventilator measures the patient's volume and rate of inspiratory gas flow, and then applies pressure support in proportion to the patient's inspiratory effort. In this mode of ventilation, inspired oxygen and positive end-expiratory pressure are manually adjusted by the clinician. In adaptive support ventilation, tidal volume and respiratory rate are automatically adjusted [7]. The patient's respiratory pattern is measured point wise in time and fed back to the controller to provide the required (target) tidal volume and patient respiratory rate. Adaptive support ventilation does not provide continuous control of minute ventilation, positive end-expiratory pressure, and inspired oxygen, these parameters need to be adjusted manually.

Smart Care ventilation monitors tidal volume, respiratory rate, and end-tidal pressure of CO₂ to maintain the patient in a respiratory “comfort” zone by automatically adjusting the level of pressure support [8]. Smart Care ventilators do not account for patient respiratory variations and do not generally guarantee adequate minute ventilation during weaning. Neurally adjusted ventilation is fundamentally different from the aforementioned automatic ventilation technologies in the sense that it uses the patient’s respiratory neural drive as a measurement signal to the ventilator [9]. In this mode of ventilation, rather than controlling pressure, the patient’s respiratory neural drive signal to the diaphragmatic electromyogram is controlled using electrodes placed on an esophageal catheter [10]. Even though this approach has been shown to be effective in some recent clinical studies [11], its effectiveness is affected if the patient is highly sedated. In addition, as in the aforementioned ventilator technologies, positive end expiratory pressure and inspired oxygen need to be manually controlled.

The common theme in modern ventilation control algorithms is the use of pressure-limited ventilation while also guaranteeing adequate minute ventilation. One of the challenges in the design of efficient control algorithms is that the fundamental physiological variables defining lung function i.e., the resistance to gas flow and the compliance of the lungs units are not constant but rather vary with lung volume. More simply, lung volume is a nonlinear function of driving pressure. In addition, these physiological variables vary from patient to patient, as well as within the same patient under different conditions, making it very challenging to develop models and effective control law architectures for active mechanical ventilation. By considering this drawback an efficient pressure and work limited neuroadaptive controller with actuator saturation and an integral constraint was designed to control lung volume and minute ventilation with input pressure constraints that also accounts for spontaneous work of breathing by the patient. But in this mode of control algorithm sensor noise, computational complexity, efficiency of online optimization, effect of control decision in cost function and constrains are not taken into the account. Moreover it doesn’t provide any evidence for control action in the case of fault in instruments which is using the particular neuroadaptive control algorithm.

In this paper, we develop nonlinear model predictive controller enhanced by a neural network optimization by considering the above mentioned cons to control the physiological characteristics of the patients which varies with applied control pressure constrains in terms of compliance and airway resistance.

The paper is organized as follows. In section II, we relate the mathematical preliminaries of nonlinear dynamic system with physiological characteristics of multi compartment lung model for a pressure limited respirator. In section III, we focus on designing the controller. Finally in section V we draw conclusion.

II. Correlation Between Mathematical Preliminaries And Physiological Characteristics of Multicompartment Lung Model

Nonlinear system is nothing but the system that is affected by a function of time as well as applied input. Similarly every compartment of a lung exhibits various nonlinear responses as a function of applied input and time, which make the breathing mechanism instability. Several definitions and some key results concerning nonlinear nonnegative dynamical systems are drawn from the reference [12], [13] that are necessary for developing the main results of this paper. Specially, for $x \in \mathbb{R}^n$ we write $x \geq 0$ to indicate that every component of X is nonnegative. Likewise, $A \in \mathbb{R}^{n \times m}$ is nonnegative or positive, if every entry of A is nonnegative or positive respectively, which is written as $A \geq 0$ or $A >> 0$ respectively.

In this paper we consider controlled nonlinear dynamical systems of the form

$$X'(t) = f(x(t)) + G(x(t))u(t), x(0) = x_0, t \geq 0 \quad (1)$$

where $x(t) \in \mathbb{R}^n, t \geq 0, u(t) \in \mathbb{R}^m, t \geq 0, f: \mathbb{R}^n \rightarrow \mathbb{R}^n$ is continuous and satisfies $f(0) = 0, G: \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m}$ is continuous, and $u: [0, \infty) \rightarrow \mathbb{R}^m$ is measurable and locally bounded. For the nonnegative system (1), we assume that $f(\cdot), G(\cdot)$ and $u(\cdot)$ satisfy sufficient regularity conditions such that has a unique solution forward in time.

The controlled nonlinear uncertain dynamical equation for a plant model G given by

$$X'(t) = A_0 x(t) + f(x(t), h(u(t)), \theta(t)) + B_0 h(u(t)), x(0) = x_0, t \geq 0 \quad (2)$$

$$y(t) = Cx(t) \quad (3)$$

where $x(t) \in \mathbb{R}^n, t \geq 0$, is the state vector, $u(t) \in \mathbb{R}^m, t \geq 0$, is the control input, $y(t) \in \mathbb{R}^m, t \geq 0$, is the system output, $A_0 \in \mathbb{R}^{n \times n}$ is a nominal known Hurwitz and essentially nonnegative matrix, $B \in \mathbb{R}^{n \times m}$ is a known nonnegative input matrix $\in \mathbb{R}^{m \times m}$ is an *unknown* nonnegative and positive definite matrix, $h(u(t)) = [h_1(u_1(t)), \dots, h_m(u_m(t))]^T$ is the constrained control input given by

$$h_i(u_i) \triangleq \begin{cases} 0, & \text{if } u_i \leq 0. \\ u_i^*, & \text{if } u_i \geq u_i^*. \\ u_i, & \text{otherwise} \end{cases} \quad (4)$$

Where $u_i^* > 0, i = 1 \dots m$, are given constants.

For the mechanical ventilation problem, the control input $u(t), t \geq 0$, represents the pressure input to the ventilator and the control input constraint (4) captures pressure amplitude limitations. In order to account lung muscle activity of the patient for the effect of spontaneous breathing the function $f(x(t), h(u(t)), \theta(t))$ is relative to pmusc using the following equation

$$f(x, h(u), \theta) = [\theta(A_{in}(x) - A_0) + (1 - \theta)(A_{ex}(x) - A_0)]x + [\theta(B_{in} - B_0) + (1 - \theta)(B_{ex} - B_0)] \times [h(u) + P_{musc}(e^T x) + P_{ex}]. \quad (5)$$

To analyze the effect of dynamic behavior of a multi compartment respiratory system in response to an arbitrary applied inspiratory pressure, we extend the linear multicompartment lung model of [12] and assume that the bronchial tree has dichotomy architecture [13], that is, in every generation each airway unit branches in two airway units of the subsequent generation. In addition, we assume that lung compliance is a nonlinear function of lung volume. For developing the state equations for inspiration and expiration for a 2^n -compartment model, where $n \geq 0$. In this model, the lungs are represented as 2^n lung units which are connected to the pressure source by n generations of airway units, where each airway is divided into two airways of the subsequent generation leading to 2^n compartments (see Fig. 1 for a four-compartment model). For our simulation we have considered only 2-compartment.

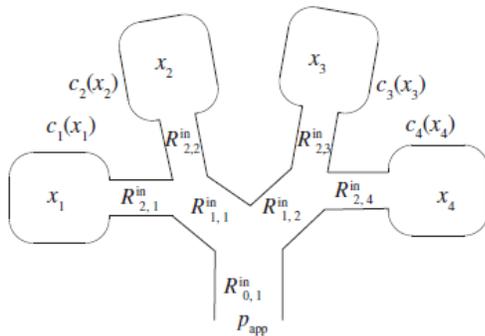


Fig. 1. Four-compartment lung model

Now, the state equations for inspiration are given by

$$R_{n,i}^{in} x_i'(t) + \frac{1}{C_i^{in}(x_i(t))} x_i(t) + \sum_{j=0}^n R_{j,kj}^{in} \times \sum_{l=(k_j-1)2^{n-j}+1}^{k_j 2^{n-j}} x_l'(t) = p_{in}(t), x_i(0) = x_{i0}^{in}, \quad 0 \leq t \leq T_{in}, i=1, \dots, 2^n \quad (6)$$

Where $C_i^{in}(x_i)$, $i = 1, 2, \dots, 2^n$ are nonlinear functions of x_i .

$i = 1, 2, \dots, 2^n$, given by

$$C_i^{in}(x_i) \triangleq \begin{cases} a_{i1}^{in} + b_{i1}^{in} x_i, & \text{if } 0 \leq x_i \leq x_{i1}^{in}, \\ a_{i2}^{in}, & \text{if } x_{i1}^{in} \leq x_i \leq x_{i2}^{in}, \\ a_{i3}^{in} + b_{i3}^{in} x_i, & \text{if } x_{i2}^{in} \leq x_i \leq TV_i \end{cases} \quad (7)$$

Now, the state equations for expiration are given by

$$R_{n,i}^{ex} x_i'(t) + \frac{1}{C_i^{ex}(x_i(t))} x_i(t) + \sum_{j=0}^n R_{j,kj}^{ex} \times \sum_{l=(k_j-1)2^{n-j}+1}^{k_j 2^{n-j}} x_l'(t) = p_{ex}(t), x_i(0) = x_{i0}^{ex},$$

$$T_{in} \leq t \leq T_{in} + T_{ex}, i=1, \dots, 2^n \quad (8)$$

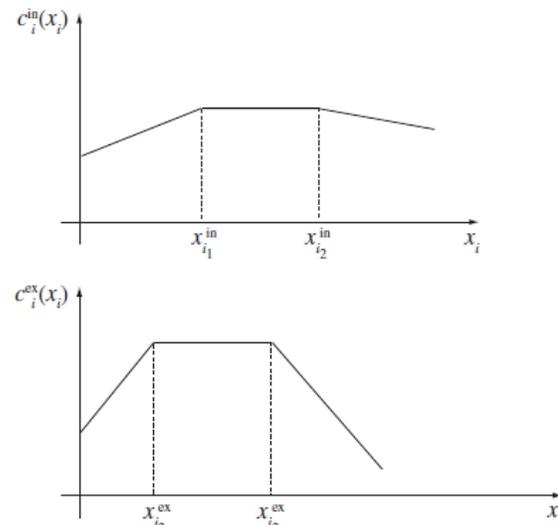


Fig. 2. Typical inspiration and expiration compliance functions as function of compartmental volumes.

Where $C_i^{ex}(x_i)$, $i = 1, 2, \dots, 2^n$ are nonlinear functions of x_i .

$i = 1, 2, \dots, 2^n$, given by

$$C_i^{ex}(x_i) \triangleq \begin{cases} a_{i1}^{ex} + b_{i1}^{ex} x_i, & \text{if } 0 \leq x_i \leq x_{i1}^{ex}, \\ a_{i2}^{ex}, & \text{if } x_{i1}^{ex} \leq x_i \leq x_{i2}^{ex}, \\ a_{i3}^{ex} + b_{i3}^{ex} x_i, & \text{if } x_{i2}^{ex} \leq x_i \leq TV_i \end{cases} \quad (9)$$

III. Controller Design

Key characteristics of the proposed approach include the use of a mathematical view of a multicompartment lung model, the capability of handling nonlinear dynamic behaviors of each compartment in terms of compliance and airway resistance for an applied input pressure. The main theme of NMPC with neural network optimization is to improve the performance in terms of MAE (%) by the feature of online selection of control pressure. Online optimization is mainly consider for handling constrain function since under or over control of input pressure will cause highly instability and produce undesirable effect to the spontaneous breathing of mechanically ventilated patients. Moreover the main highlighting point of NMPC is that it has an ability to control the future state of the plant model and henceforth its efficiency in controlling the respiratory drive of the critical care patients

can be improved. Performance of MPC have been proposed in the area of handling unmanned aircraft [14]. The operation of the NMPC with neural network optimization can be divided into two parts; the prediction where the future state sequence is calculated based on the initial control sequence depends on the initial state of the plant model and during optimization where the control sequence is updated to reduced the error function $L(x, u)$. This process is repeated using the updated control sequence until an optimal sequence is obtained.

Putting together the neural network optimization with the traditional nonlinear model predictive controller, we arrive at the NMPC/Neural network optimization that was used in this paper.

A. Prediction

In order to achieve output tracking, we construct a reference nonnegative dynamical system of G_{ref} and it is given by

$$\begin{aligned} x_{ref}(t) &= A_{ref} x_{ref}(t) + B_{ref} r(t), x_{ref}(0) = x_{ref0}, t \geq 0 \quad (10) \\ y_{ref}(t) &= C x_{ref}(t). \quad (11) \end{aligned}$$

Initial state of the system can be refer from (12) and (13) where $x_{ref}(t) \in \mathbb{R}^n, t \geq 0$, is the reference state vector, $r(t) \in \mathbb{R}^d, t \geq 0$, is a bounded piecewise continuous nonnegative reference input, $A_{ref} \in \mathbb{R}^{n \times n}$ is a Hurwitz and essentially nonnegative matrix, and $B_{ref} \in \mathbb{R}^{n \times d}$ is a nonnegative matrix Status of the plant model at $T=t+1$ depends on past status of the plant model i.e., at $T=t$ state, which can be expressed as

$$X'(t+1) = A_0 x(t) + f(x(t), h(u(t)), \theta(t)) + B_0 h(u(t)), \quad t \geq 0 \quad (12)$$

$$y(t+1) = Cx(t) \quad (13)$$

Where $A_0 = -R_{av}^{-1} C_{av}$, $B_0 = R_{av}^{-1} e$, and $C = e^T$, and R_{av} and C_{av} are nominal parameter matrices given by

$$R_{av} \triangleq \sum_{j=0}^n \sum_{k=1}^{2^j} R_{j,k}^{av} z_{j,k} z_{j,k}^T, \quad C_{av} \triangleq \text{diag} \left[\frac{1}{c_{1}^{av}}, \dots, \dots, \frac{1}{c_{2^n}^{av}} \right]$$

Sampling eqn (12),(13) using Taylor series and truncate the first term.

$$X(t+1) = A_0 x(t) + t_s f(x(t), h(u(t)), \theta(t)) + B_0 h(u(t)), \quad t \geq 0. \quad (14)$$

$$y(t+1) = C t_s g(x(t)) u(t) \quad (15)$$

Where $x(t) \in \mathbb{R}^n, t \geq 0$

The output response of the plant for an applied control sequence is compared with the reference model of the plant, thus the cost function is estimated and this variation in cost function will reflect its effect to the constrain function in terms of lung resistance and compliance uncertainty. In discrete domain it can be viewed in form of variation in steady state of constrain function. This can be shown in the equation given below:

$$\begin{aligned} \min_{u(i) \in U} \sum_{i=0}^{t_f} L(x(i), u(i)) \text{ s.t. } C_j(u, x) \leq 0 \\ \forall j \in [0, N_r] \end{aligned} \quad (16)$$

The use of $C(x, u) \leq 0$ does not impose any, restriction on the form of constraints, but is rather used to allow direct mapping to the way constraints are handled by the neural network. The future state sequence can be obtained directly from (11). Nevertheless the value of dL/du must also be calculated to be used for optimization.

$$\frac{dx(k+1)}{dh(u(j))} = \left[I + B_0 h(u(t)) + t_s J_f \left(x(k), h(u(k)), \theta(k) \right) \right] \times \frac{dx(k)}{dh(u(j))} \quad (17)$$

$$\frac{dy(k+1)}{dh(u(j))} = C t_s g(x(k)) \frac{dh(u(k))}{dh(u(j))} \quad (18)$$

Where $h(u(i))$ can be obtained by the control input according to the condition specified in (4).

Since the action affect only the future actions

$$\frac{dx(k)}{dh(u(i))} = 0 \quad \forall x(k), u(j): j \geq k \quad (19)$$

and do not depend on past or future actions

$$\frac{dh(u(k))}{dh(u(j))} = \begin{cases} 1, & \text{if } j = k \\ 0, & \text{otherwise.} \end{cases} \quad (20)$$

Using (18), (19) and taking into account (20) and (21), the following update rule for calculating the $dx(t)/dh(u(j))$ from previous prediction step is obtained.

$$\frac{dx(k+1)}{dh(u(j))} = \begin{cases} 0, & \text{if } k+1 \\ t_s g(x(k)), & \text{if } k+1 = j \\ K(k) \frac{dx(k)}{du(i)}, & \text{otherwise.} \end{cases} \quad (21)$$

Where $K(k) = I + B_0 h(u(k)) + t_s J_f \left(x(k), h(u(k)), \theta(k) \right)$

A cost function that separates the contribution of the control sequence from that state of the patient's respiratory system is assumed

$$\frac{dL}{dh(u(j))} = \sum_{i=1}^{N_s} L^* x(i) + w u^T u. \quad (22)$$

Where L^* is a function of the state and w is positive weight factor and vector $x \triangleq [x_1, x_2, \dots, x_{2^n}]^T$.

The derivative of cost function is given by

$$\frac{dL}{du} = \sum_{i=1}^{N_s} \left(\frac{\partial L^*}{\partial x(i)} \frac{dx(i)}{du} \right) + 2wu. \quad (23)$$

B. Nonlinear Optimization.

As discussed in section IV-B, the NMPC optimization problem can be expressed by

$$\min_{h(u(i)) \in h(U)} \sum_{i=0}^{tf} L(x(i), h(u(i))) \text{ s.t } C_j(h(u), x) \leq 0$$

Where L is the cost function and c an appropriate expression of the constraints imposed. To solve the above mentioned problem in online, neural network optimization is used. The specialty feature this optimization lies in solving convex, nonlinear optimization problems with linear or nonlinear constraints. The idea behind this approach is to build a neural network that maps an ODB whose equilibrium point is the optimal solution to the problem(11),(12) and (13).A significant advantage of this approach is that, it does not require calculation of Hessian matrix of Langragian.

C.Integration.

Putting together the neural network optimization with the traditional nonlinear model predictive controller, we arrive at the NMPC/Neural network optimization that was used in this paper.

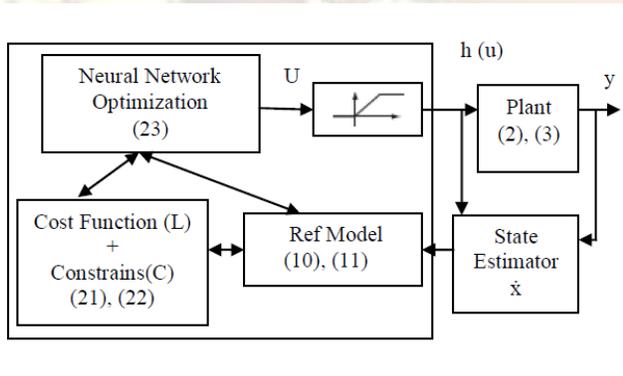


Fig. 3. Block diagram of NMPC with neural network optimization

Algorithm for NMPC with Neural Network Optimization.

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for each New sensor data (plant output y) received do outer loop
Filter(y) to obtain an estimate on current state in terms
of compartment volume  $x(0) = [x_1 \dots x_n]^T$ 
Estimate  $h(u(i))$  for an initial control pressure input
 $u(i)$ .
for  $k \leftarrow 0$  to  $E-1$  do Inner loop
 $\frac{dL}{dh(u(j))} \leftarrow wu^T u$ .
for  $i \leftarrow 0$  to  $N_s$  do
for  $j \leftarrow 0$  to  $N_c-1$  do
if  $i = j$  then
 $\left| \frac{dx(i)}{dh(u(j))} \leftarrow t_s g(x) \right.$ 
else if  $i > j$  then
 $\left| \frac{dx(i)}{dh(u(j))} \leftarrow K(i) \frac{dx(i-1)}{dh(u(j))} \right.$ 
end
end
 $x \leftarrow A_0 x + B_0 h(u) + t_s f(x, h(u), \theta)$ 
 $y \leftarrow C t_s g(x) h(u)$ 
 $\frac{dL}{dh(u(j))} \leftarrow \frac{dL}{dh(u(j))} + \frac{dL}{dx} \frac{dy}{dh(u(j))}$ 
Calculate C
for  $j \leftarrow 1$  to  $N_c - 1$  do
Calculate  $\frac{dc}{dh(u(j))}$ 
end
 $\chi \leftarrow \chi + \gamma [-\chi + (\chi - C(u))^+]$ 
 $u = u + \gamma \left[ -u + \left( u - \frac{dL}{du} - \frac{dc}{du} \chi \right)^+ \right]$ 
end
Send control command  $u(0)$ 
 $u \leftarrow [u(1), u(2) \dots \dots \dots u(N_c-1)]$ 
end.
    
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IV. Simulation Results

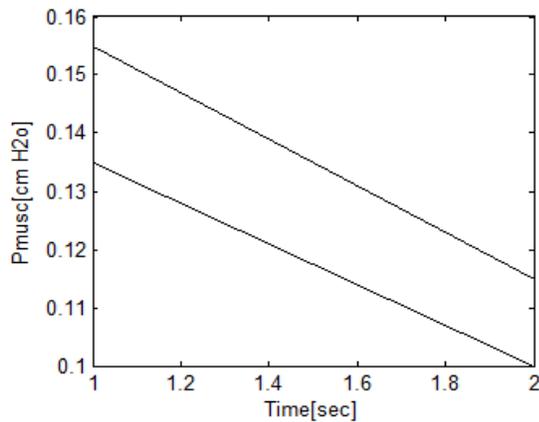


Fig. 4. Status of respiratory muscles for non sedative condition.

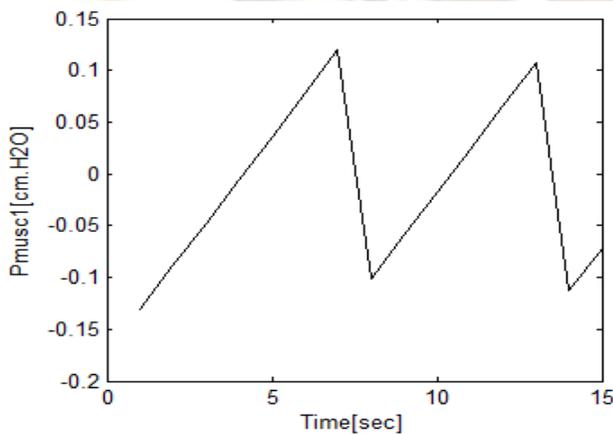


Fig. 5. Status of respiratory muscles for sedative condition due to compartment volume1.

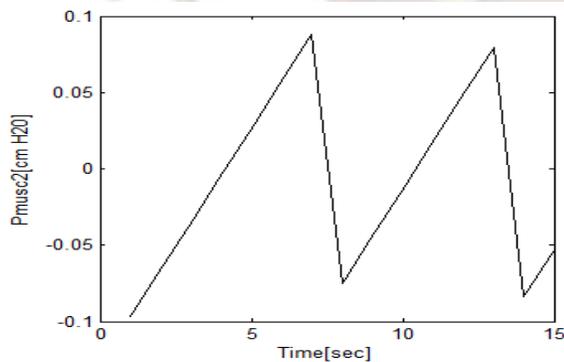


Fig. 6. status of respiratory muscles for sedative condition due to compartment

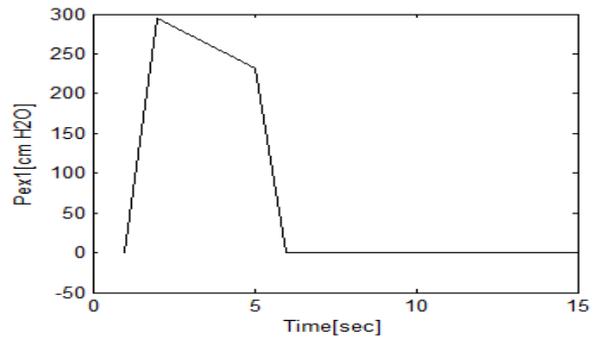


Fig. 7 variation of peep for the compantment1

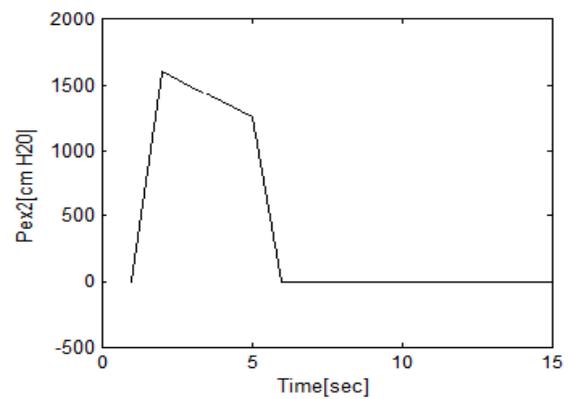


Fig. 8 variation of PEEP for the compantment2

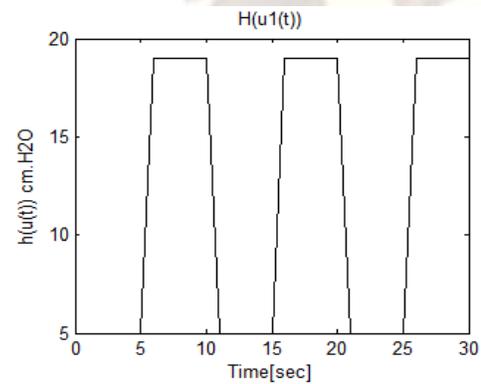


Fig. 9 Constrained pressure $P(t) = h(u(t))$ Versus time.

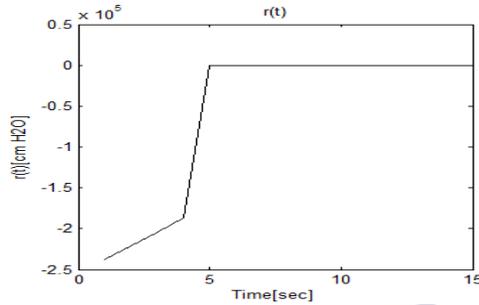


Fig.10 Variation in respiratory parameter for a single cycle.

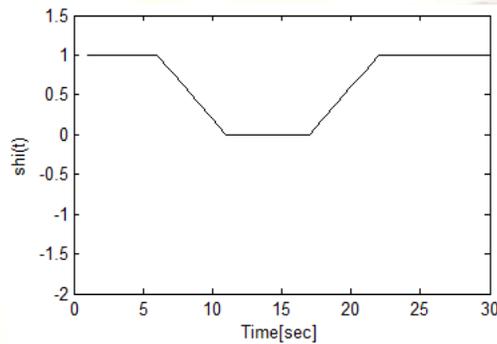


Fig. 11 Effect of saturation impact time for the compartments

V. Conclusion

From the above simulation results, we conclude that respiratory parameter such as lung volume and compliance plays a remarkable role in the life of respiratory failure patients. If the input pressure increases volume will increase, consequently PEEP will also get increases in nonlinear manner. This is due to the fact that the gas molecules in applied pressure without the consideration of compartmental volume will strike each chamber in different manner and the force of exertion due to this interaction will cause instability in the breathing condition of the patients. By considering these drawbacks a mathematical view of a lung model is designed and various physiological parameters of such lung model are analyzed for applying the control input with an saturation constrain limit for the patients under critical conditions. Due to the fact that a physiological characteristic of the respiratory system varies from patients to patients, convex problem of decision making will arise in supplying the control input. This can be overcome by a controller action. By relating the human nervous system with the new technology world an innovation of neural network has been take place. The remarkable features of the neural network provide a chance to implement NMPC with neural network optimization in the field medical systems and finally the performance is improved by optimizing the cost and constrain function in terms of derivative and hence forth minute error can be minimized and better choice of saturated pressure values can be chosen for

controlling the compliance and resistance of the compartment volume.

APPENDIX

$\theta(t)$ Continuous transition of the lung resistance and Compliance parameters between inspiration and expiration.

a_{ij}^{in} j =1, 2, 3. Compliance parameter during inspiration.

b_{ij}^{in} j =1, 2, . Compliance parameter during inspiration.

a_{ij}^{ex} j =1, 2, 3. Compliance parameter during expiration.

b_{ij}^{ex} j =1, 2, 3. Compliance parameter during expiration.

$\epsilon_{in} > 0$
 $\epsilon_{ex} > 0$ small constants representing the transition times form inspiration to expiration.

u_i^* Peak Pressure Limit.

$X'(t)$ Nonlinear Function of lung model at time t.

$x(t)$ Volume of the Compartment in the lung at time t.

$u(t)$ Control input to the ventilator at time t.

$y(t)$ output of the plant at time t.

C Constraint function.

γ Learning Rate.

N_c Control Horizon

N_s Prediction Horizon

N_γ Number of constraints

χ Supporting Vector.

t_s Sampling Period.

PEEP Positive End Expiratory Pressure.

A_0 nominal parameter matrices

$C_{in}(t)$ Nonlinear function defining the lung compliance during inspiration.

B nominal parameter matrices

C nominal parameter matrices.

$C_{in}(t)$	Nonlinear function defining the lung compliance during inspiration.
$C_{ex}(t)$	Nonlinear function defining the lung compliance during expiration.
$C_i^{in}(x_i)$	Compliance due to i^{th} compartment of volume x_i during inspiration. Where $i=1,2,\dots,2^n$.
$C_i^{ex}(x_i)$	Compliance due to i^{th} compartment of volume x_i during expiration. Where $i=1,2,\dots,2^n$.
$h(u(t))$	Saturated Control input pressure.
$P_{ex}(t)$	End-expiratory pressure.
$P(t)$	Constrained Pressure.
$P_{in}(t)$	Pressure applied during inspiration.
$V_a(t)$	Alveolar ventilation.
R_{in}	Resistance of airway during inspiration time period.
R_{ex}	Resistance of airway during expiration time period
$R^{in} j,i$	Resistance of i^{th} airway unit in j^{th} generation during inspiration.
$R^{ex} j,i$	Resistance of i^{th} airway unit in j^{th} generation during expiration.
VCO_2	Total body production of CO_2 per minute.
$PaCO_2(t)$	Arterial partial pressure of CO_2 in the blood.
$TV(t)$	Tidal Volume
$RR(t)$	Respiratory Rate
T	Length of one single breath
T_{in}	Inspiratory Time.
T_{ex}	Expiratory Time.

REFERENCES

BOOKS

[1] M. J. Tobin, *Principles and practice of mechanical ventilation* (New York: McGraw-Hill, 1994).

[2] S. P. Pilbeam and J. M. Cairo, *Mechanical ventilation, physiological and clinical applications* (4th ed. St. Louis, MO: Mosby, 1987).

Chapters in Books:

[3] M. Younes, Proportional assist ventilation, in M.J.Tobin, (Ed.), *Principles and practice of Mechanical Ventilation*(New York: McGraw-Hill 1994) 349–369.

Journal papers:

[4] M. Dojat, L. Brochard, F. Lemaire, and A. Harf, A knowledge-based system for assisted ventilation of patients in intensive care units, *International Journal of Clinical Monitoring and Computing*. 1992; 9:239-250.

[5] T. P. Laubscher, W. Heinrichs, and N. Weiler, An adaptive lung ventilation controller, *IEEE Trans BiomedEng.*, vol. 41, no. 1, pp51–59, Jan. 1994.

[6] C. Sinderby, P. Navalesi, J. Beck, Y. Skrobik, N. Comtois, S. Friberg, S. B. Gottfried, and L. Lindström, Neural control of mechanical ventilation in respiratory failure, *Natural Med.*, vol. 5, no. 12 pp.1433-1436, 1999.

[9] A. Huszczuk, A respiratory pump controlled by phrenic nerve activity, *J. Physiol.*, vol. 210, no. 2, pp. 183–184, 1970.

[10] J. Barwing, M. Ambold, N. Linden, M. Quintel, and O. Moerer, Evaluation of the catheter positioning for neutrally adjusted ventilator assist, *Intens. Care Med.*, vol. 35, no. 10, pp. 1809–1814, 2009.

[11] W. M. Haddad and V. Chellaboina, Stability and dissipativity theory for nonnegative dynamical systems: A unified analysis framework for biological and physical systems,” *Nonlinear Anal.:Real World Appl.*, vol. 6, no. 1, pp. 35–65, Feb. 2005.

[13] E. R. Weibel, *Morphometry of the Human Lung*, New York: Academic, 1963.

Proceedings Papers:

[7] F. T. Tehrani, “Automatic control of an artificial respirator,” in *Proc.Int.Conf. IEEE Eng. Med. Biol. Soc.*, vol. 13. Nov. 1991, pp. 1738-1739.

[14] A. Richards and J. How, Decentralized model predictive control of Cooperating uvas. in proc .43rd IEEE conf. Decision Control (CDC’04).

[12] V. Chellaboina, W. M. Haddad, J. M. Bailey, and H. Li, Limit cycle stability analysis of a multi-compartment model for a pressure-limited respirator and lung mechanics system,” in *Proc. Amer. Control Conf.*, 2007, pp. 2024–2029.