A Novel Approach to Overcome the Intertwined Shortcomings of DWT For Image Processing and De-noising

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ABSTRACT
Shift variance, poor directional selectivity, oscillations and aliasing are four fundamental, intertwined shortcomings of the DWT that undermines its application for certain image processing tasks. The initial motivation behind the work to use advanced CWT which will overcome the limitations of standard DWT. In this paper, we demonstrate that excellent shift-invariance properties and directional selectivity with transform-domain redundancy in 2D. We achieve this by projecting the wavelet coefficients from Selesnick's almost shift-invariant, double-density wavelet transform so as to separate approximately the positive and negative frequencies, thereby increasing directionality. Subsequent decimation and a novel inverse projection maintain the low redundancy while ensuring perfect reconstruction. Although the advanced CWT generates complex-valued coefficients allowing processing capabilities that are impossible with real-valued coefficients. The proposed method is highly efficient and useful for image processing applications and also outperforms the method proposed by the previous authors for image de-noising with significant improvement in the value of PSNR over RMSE as given by the other classical methods.

KEYWORDS- Complex DWT, De-noising, 1-D DWT, 2-D DWT, Redundant CWT.

1. INTRODUCTION

Many scientific experiments results in a datasets corrupted with noise, either because of the inadequate data acquisition process, or because of environmental effects. A first preprocessing step in analyzing such datasets is de-noising, that is, removing the unknown signal of interest from the available noisy images. There are several approaches to de-noise images. Despite similar visual effects, there are subtle differences between de-noising, de-blurring, smoothing and restoration.

It has been observed that standard DWT and its extensions suffer from two or more serious limitations. The initial motivation behind the earlier development of complex-valued DWT was the third limitation that is the 'absence of phase information'. Complex Wavelets Transforms(CWT) use complex-valued filtering(analytic filter) that decomposes the real/complex signals into real and imaginary parts in transform domain. The real and imaginary coefficients are used to compute amplitude and phase information, just the type of information needed to accurately described the energy localization of oscillating functions(wavelet basis).

Edges and other singularities in signal processing applications manifest themselves as oscillating coefficients in the wavelet domain. The amplitude of these coefficients describes the strength of the singularity while the phase indicates the location of singularity. In order to determine the correct value of localized envelop and phase of an oscillating function, 'analytic' or 'quadrature' representation of the signal is used. This representation can be obtained from the Hilbert transform of the signal. Thus, the complex orthogonal wavelet may prove to be a good choice, since it will allow processing of both magnitude and phase simultaneously.

This paper presents the concept of DT-DWT versions of RCWT have limited redundancy with very good properties of shift-invariance, improved directionality and availability of phase information, which are not present in standard DWT. RCWT has a huge potential in signal/image de-noising.. The paper is organize as follows: Section 2 involves Separable DWT. The Complex Dual Tree DWT is discussed in Section 3 & Section 4 deals with bivariate shrinkage functions. Section 5 gives the general method involves in image de-noising. Section 6 deals with results & discussions & the conclusion are given in Section 7.

2. Separable DWT

2.1 1-D Discrete Wavelet Transform

In separable DWT the analysis filter bank decomposes the input signal x(n) into two sub band signals, c(n) and d(n). The signal c(n) represents the low frequency part of x(n), while the signal d(n) represents the high frequency part of x(n). We denote the low pass filter by af1(analysis filter 1) and the high pass filter by af2(synthesis filter 2). As depicted in figure(1), the output of each filter is then down sampled by 2 to obtain the two sub band signals c(n) & d(n).

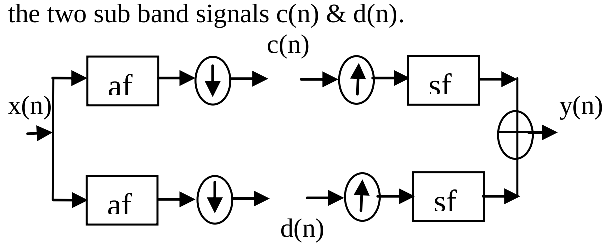


Fig.(1): Separable DWT(Analysis) & (Synthesis) Filter Bank

The Synthesis filter bank combines the two sub band signals c(n) & d(n) to obtain a single signal y(n). The synthesis filter bank up-samples each of the two sub band signals. The signals are then filtered using a low pass and high pass filter. We denote the low pass filter by sf1(synthesis filter 1) and the high pass filter by sf2(synthesis filter 2). The signals are then added together to obtain the signal y(n). If the four filters are designed so as to guarantee that the output signal y(n) equals the input signal x(n), then the filters are said to satisfy the perfect reconstruction condition.

2.2 2-D Discrete Wavelet Transform

image-processing applications requires two-dimensional implementation of wavelet transform. Implementation of 2-D DWT[3],[4],[5] is also referred to as 'multidimensional' wavelet transform in literature. In the 2D case, the 1D analysis filter bank is first applied to the columns of the image and then applied to the rows. If the image has N1 rows and N2 columns, then after applying the 1D analysis filter bank to each column we have two sub band images, each having N1/2 rows and N2 columns; after applying the 1D analysis filter bank to each row of both of the two sub band images, four sub band images are obtained, each having N1/2 rows & N2/2 columns. This is depicted in figure (2) given below. The 2D synthesis filter bank combines the four sub band images to obtain the original image of size N1 by N2 [4][5].

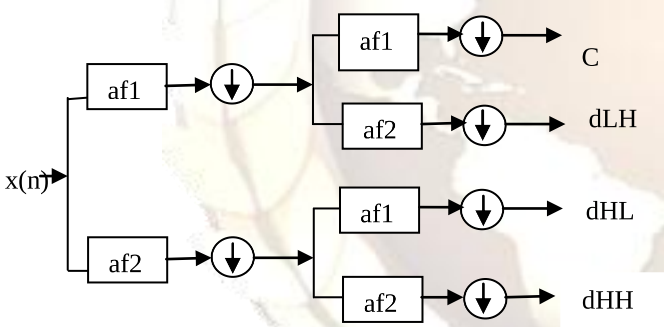


Fig.(2): One stage in multi-resolution wavelet decomposition of an image x(n)

3. The Complex Wavelet Transform:

There is no one unique extension of the standard DWT into the complex plane. The complex valued symmetric Daubenchies Wavelets (SDW) has been used for applications such as image enhancement, restoration and coding. The recent research in the field of CWT[1],[2] is directed towards the design of complex valued filter bank structure such that the resulting wavelets(real and imaginary parts) after high pass filtering from the Hilbert transform pairs at each successive level in the framework of standard DWT decomposition structure.

Recent development of CWTs can be broadly classified in two groups; RCDWT(Redundant CWTs) and NRCWT(Non-redundant CWTs). Standard DWT is critically decimated and gives N samples in transform domain for the same N samples of a given signal. While the redundant transform gives M samples in transform domain for N samples of given input signal (where M>N) and hence it is expensive by the factor M/N. The RCWT include two almost similar CWTs. They are denoted as DT-DWT(Dual-Tree DWT based CWT) with two almost similar versions namely Kingsbury's DT-DWT(K), and Selesnick's DT-DWT(S). These redundant transforms consist of two conventional DWT filter bank trees working in parallel with respective filters of both the trees in approximate quadrature. The filter bank structure of both DT-DWTs is same but the design methods to generate the filter coefficients are different. Both DT-DWTs provide phase information; they are shift-invariant with improved directionality. Selesenik proposed an alternative filter design methods for DT-DWT(K) and designed DT-DWT(S), almost equivalent to DT-DWT(K) such that in the limit the scaling and wavelet functions form Hilbert transform pairs. DT-DWT(S) is designed with simple methods to obtain filter coefficients.

4. Redundant Complex Wavelet Transform

RCWT comprises of two types of DT-DWT (Dual-Tree DWT based complex wavelet transforms) one is Kingsbury's DT-DWT(K) and the other is Selesnick's DT-DWT(S). These DT-DWT based transforms are redundant because of two conventional DWT filter bank trees working parallel and are interpreted as complex because of the respective filters of both the trees are in approximate quadrature. In other words, respective scaling and the wavelet functions at all decomposition levels of both the trees form the (approximate) Hilbert transform pairs. Both versions of DT-DWT use 2-band perfect reconstruction filter sets.

The DT-DWT(S) is the less redundant version of its primitive 3-band perfect reconstruction Double-Density Dual-Tree DWT(DDTWT) structure that can e found. The DDTWT is derived by combining two parallel Double-Density DWT(DDWT) trees, preserving the quadrature properties of their respective wavelet filters. Thus, DDTWT has two important benefits: first is the shift-invariance due to redundant DDWT and second is Hilbert pair of wavelets because of two parallel trees in quadrature. It is important to note that DDWT is a shift invariant 3-band redundant DWT but not a CWT. Both DT-DWTs have same filter bank structure. The DT-DWTs use the analysis and synthesis filter bank structures the seems identical to those used for standard DWT as shown in figures() and (). The key difference is that all the real filters are replaced with analytic filters formulated in figure() to have complex solutions. The replacement of real filters with analytic filters makes the new structure equivalent to two standard DWT filter bank structures operating in parallel. Because of two parallel trees for analysis and synthesis, these CWT are described as Dual-Trees DWT based CWT. The insertion of the parallel structure eliminates the disadvantages of standard DWT.

4.1 Selesnick's DT-DWT based CWT:

The filter bank structures for both DT-DWTs are identical. Figures(3) and (4) shows 1-D analysis and synthesis filter banks spanned over three levels. It is evident from the filter bank structure of DT-DWT that is resembles the filter bank structure of standard DWT with twice the complexity. It can be seen as two standard DWT trees operating in parallel. One tree is called as a real tree and other is called as an imaginary tree. Sometimes in future discussions the real tree will be referred to as tree-a and the imaginary tree as tree-b.

The form of conjugate filters used in 1-D DT-DWT is given as: (h_x+j_x)

Where, h_x is the set of filters $\{h_0,h_1\}$, and g_x is the set of filters $\{g_0,g_1\}$both sets in only x-direction(1-D).

The filters h_0 and h_1 are the real-valued low pass and high pass filters respectively for real tree. The same is true for g_0 and g_1 for imaginary tree.

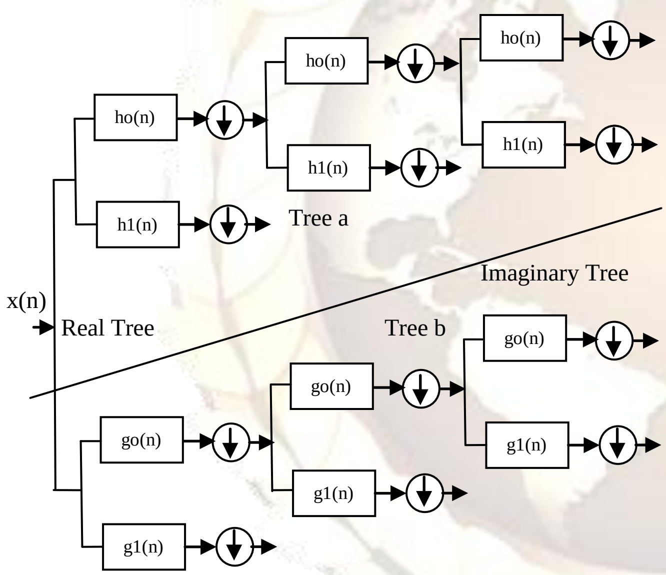
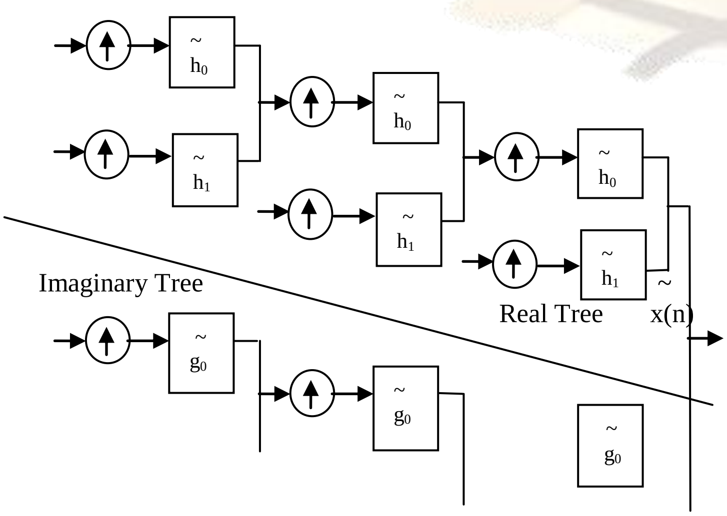


Fig.(3): Analysis Filter Bank for 1-D DT- DWT



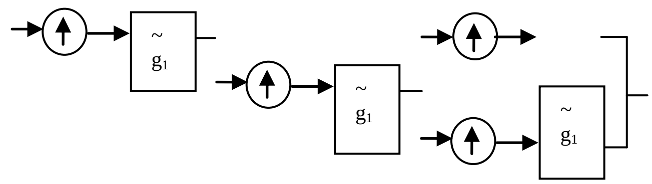


Fig.(4): Synthesis Filter Bank for 1-D DT- DWT

Though the notation of h_0 and h_1 are use for all level in the real part of analysis tree, h_0 and h_1 of first level are numerically different then the respective filters at all other levels above level-1. The same notation is applied for imaginary tree filters g_0 and g_1. The synthesis filter pairs Ãh_0, h_1, and g_0, g_1 as shown in figure(4) from orthogonal or bi-orthogonal pairs with their respective counterpart filters of analysis tree as shown in figure(3). The 2-D DT-DWT structure has an extension of conjugate filtering in 2-D case. The filter bank structure of 2-D dual-tree is shown in figure(5). 2-D structure needs four trees for analysis as well as for synthesis. The pairs of conjugate filters are applied to two dimensions (x and y), which can be expressed as:

$$(h_x+jg_x)\,(h_y+jg_y) = (h_xh_y-g_xg_y) + j(h_xg_y + g_xh_y)----(1)$$

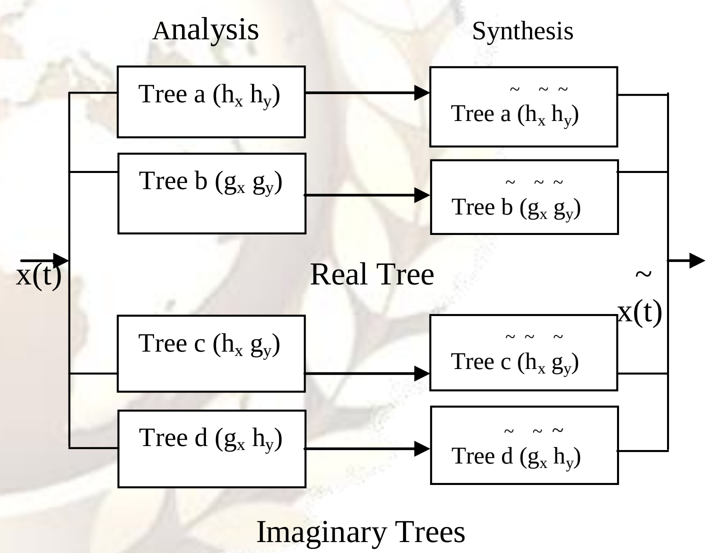


Fig.(5) Filter bank structure for 2-D DT-DWT

5. 2-D De-noising:

In this section 2-D de-noising performance of redundant CWT DT-TWT(S) is compared with other wavelet based algorithms such as standard DWT, SWT by using relevant Matlab (Image Processing toolbox) functions.

5.1 Image and Noise Model:

The standard test images such as Lena, Kaiterina kaif are taken for experiments. Original images are corrupted by additive white Gaussian noise.

The image and noise model is given as:

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$$x = s + \sigma.g \quad \text{-----------------(2)}$$

where, s is an original image and x is a noisy image corrupted by additive white Gaussian noise g of standard deviation σ. Both images s and x are of size N by M (mostly M=N and always power of 2).

5.2 Algorithm:

2-D de-noising is an extension of 1-D de-noising based on 2-D separable WT implementation. For all conventional filtering methods, 3-by-3 filter kernel is taken for convolution. For all wavelet based methods, decomposition is performed up to J levels. For standard DWT and SWT the arbitrary wavelet basis employed is 'wtype'. Filters for DT-DWT(S) ('w-type-s') are given below in Tables (1),(2),(3) and (4):

	Analysis Filters			
	Real Tree		Imaginary Tree	
	Low Pass	High Pass	Low Pass	High Pass
1	0	0	0.01122679215254	0
2	-0.08838834764832	-0.01122679215254	0.01122679215254	0
3	0.08838834764832	0.01122679215254	-0.08838834764832	-0.08838834764832
4	0.69587998903400	0.08838834764832	0.08838834764832	-0.08838834764832
5	0.69587998903400	0.08838834764832	0.69587998903400	0.69587998903400
6	0.08838834764832	0.69587998903400	0.69587998903400	-0.69587998903400
7	-0.08838834764832	0.69587998903400	0.08838834764832	0.08838834764832
8	0.01122679215254	-0.08838834764832	-0.08838834764832	0.08838834764832
9	0.01122679215254	-0.08838834764832	0	0.01122679215254
10	0	0	0	-0.01122679215254

Table 1: 1st Stage Analysis Filters

	Synthesis Filters			
	Real Tree		Imaginary Tree	
	Low Pass	High Pass	Low Pass	High Pass
1	0	0	0	0.01122679215254
2	0.01122679215254	-0.08838834764832	0	-0.01122679215254
3	0.01122679215254	-0.08838834764832	-0.08838834764832	-0.08838834764832
4	-0.08838834764832	0.69587998903400	0.08838834764832	-0.08838834764832
5	0.08838834764832	0.69587998903400	0.69587998903400	0.69587998903400
6	0.69587998903400	0.08838834764832	0.69587998903400	-0.69587998903400
7	0.69587998903400	0.08838834764832	0.08838834764832	0.08838834764832
8	0.08838834764832	0.01122679215254	-0.08838834764832	0.08838834764832
9	-0.08838834764832	-0.01122679215254	0.01122679215254	0
10	0	0	0.01122679215254	0

Table 2: 1st Stage Synthesis Filters

	Analysis Filters			
	Real Tree		Imaginary Tree	
	Low Pass	High Pass	Low Pass	High Pass
1	0.03516384000000	0	0	-0.03516384000000
2	0	0	0	0
3	-0.08832942000000	-0.11430184000000	-0.11430184000000	0.08832942000000
4	0.23389032000000	0	0	-0.08838834764832
5	0.76027237000000	0.58751830000000	0.58751830000000	0.76027237000000
6	0.58751830000000	-0.76027237000000	0.76027237000000	0.58751830000000
7	0	0.23389032000000	0.23389032000000	0
8	-0.11430184000000	-0.08832942000000	-0.08832942000000	-0.11430184000000
9	0	0	0	0
10	0	-0.03516384000000	0.03516384000000	0

Table 3: Remaining Stage Analysis Filters

Table 4: Remaining Stage Synthesis Filters

Synthesis Filters				
	Real Tree		Imaginary Tree	
	Low Pass	High Pass	Low Pass	High Pass
1	0	-0.03516384000000	0.03516384000000	0
2	0	0	0	0
3	-0.11430184000000	0.08832942000000	-0.08832942000000	-0.11430184000000
4	0	0.23389032000000	0.23389032000000	0
5	0.58751830000000	-0.76027237000000	0.76027237000000	0.58751830000000
6	0.76027237000000	0.58751830000000	0.58751830000000	-0.76027237000000
7	0.23389032000000	0	0	0.23389032000000
8	-0.08832942000000	-0.11430184000000	-0.11430184000000	0.08832942000000
9	0	0	0	0
10	0.03516384000000	0	0	-0.03516384000000

5.3 Performance Measure:

The quantitative measures for 2-D de-noising, namely MSE(Mean Square Error) and PSNR(Peak Signal to Noise Ratio) are determined as:

$$MSE = 1/NM \sum_{n=1}^{N} \sum_{m=1}^{M} [s(n,m) - y(n,m)]^2 \text{-------}(3)$$

$$PSNR = 10 \log_{10}\left(\frac{255}{\sqrt{MSE}}\right) \text{--------------------------------}(4)$$

Where, s is an original image and y is a recovered image from noisy image x. The quantitative performance is evaluated through human visual system by observing the recovered images with various algorithms.

6 Results and Discussion:

A few sample results of de-noising performance based on quantitative measure are presented in tables (5) and (6) and the quantitative performance based on human visual system for various algorithms are shown in figures(6) and (7). The effect of improved directionality on de-noising for redundant CWT DT-DWT compared to less directional standard DWT is quite clear for 'pattern' image for all noise conditions. For human visual perspective, the performance of various algorithms for high noise conditions is quite clear. The performance of DT-DWTs is distinguishably superior to standard DWT. But under low noise conditions, minute differences are very difficult to perceive hence all wavelet based methods seem to have nearly same visual effects.

Performance Measure for De-noising Methods	Image with σ = 10				
	Lena	Goldhill	Oeppers	Parrot	Katrina
Initial MSE	100	100	100	100	100
	MSE (Hard Thresholding)				
Standard DWT	56	65	43	45	79
SWT	39	41	28	26	42
DT-DWT(S)	35	38	26	23	30
	MSE (Soft Thresholding)				
Standard DWT	44	48	38	40	63
SWT	36	39	28	28	47
DT-DWT(S)	33	37	27	27	38

Table (5): MSE for various de-noising methods (σ=10) for different images: (a) Hard thresholding (b) Soft tthresholding with parameters J=3, stpsz = 1, wtype = 'db2', wtype-s = 'FSfilt'.

Performance Measure for De-noising Methods	Image with σ = 10				
	Lena	Goldhill	Oeppers	Parrot	Katrina
Initial PSNR in dB	28.12	28.15	28.12	28.12	28.15
	(a) PSNR in dB (Hard Thresholding)				
Standard DWT	30.63	29.99	31.71	31.52	29.15
SWT	32.24	31.95	33.65	33.93	31.88
DT-DWT(S)	32.64	32.25	33.84	34.38	33.28
	(b) PSNR in dB (Soft Thresholding)				
Standard DWT	31.65	31.28	32.23	32.10	30.14
SWT	32.59	32.18	33.60	33.53	31.39
DT-DWT(S)	32.87	32.33	33.77	33.76	32.27

Table(6): PSNR for various de-noising methods (σ=10) for different images: (a) Hard Thresholding (b) Soft Thresholding with parameters J = 3, stpsz = 1, wtype='db2', wtype-s='FSfilt' and 'otherfilt'.

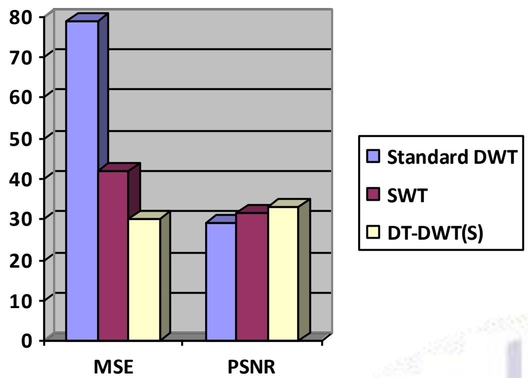

Fig.(6):MSE Vs Peak Signal-to- Noise Ratio(Hard Thresholding)

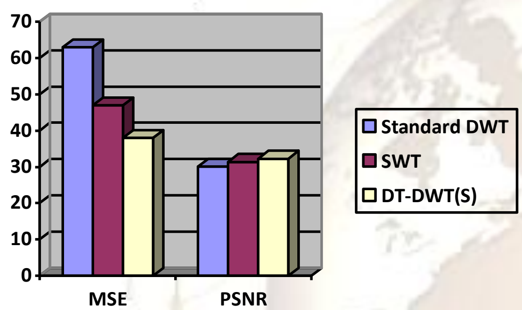

Fig.(7):MSE Vs Peak Signal-to- Noise Ratio(Soft Thresholding)

6.1 Results via Soft Thresholding


Original Image **Noisy Image**


Separable DWT **DT-DWT(S)**

Fig.(8): Output Results via Soft Thresholding

6.2 Results via Hard Thresholding


Separable DWT **DT-DWT(S)**

Fig.(9): Output Results via Hard Thresholding

The de-noised image obtained using hard and soft thresholding has a PSNR of 33.28 and 32.27 in dB respectively. The de-noised image obtained using conventional wavelet transform has a PSNR of 29.15 dB and 30.14 dB for Hard and Soft thresholding. Thus the Redundant CWT type DT-DWT(S) gives better performance over the classical method. The values are tabulated above in table(6).

7 Conclusion and Future Scope

In this paper, a advance complex wavelet transform such as Redundant DT-DWT(S) is proposed for image de-noising. This new rule maintains the simplicity, efficiency and classical soft thresholding approach. The result is simulated on Matlab 7.0.1 environment. The Simulation results of Katrina Kaif for classical Separable DWT and Complex Dual-Tree DWT are shown in above figures(8) and (9). With these results it is clear that the proposed method gives significant improvement in terms of image quality and preserve the useful information from the original image. These properties are important for many applications in image processing. In future, this work can be extended by using different types of wavelets for different values of noise variance also.

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