Pre-Optimization of Brushless PM Motor Design using Interval Arithmetic

S. Sashidhar[#], GautamMoni Patro*, Venkata R Basam[$], P. Mallikarjuna Rao[@]

PG Scholar,
[#]Department of Electrical Engineering, Andhra University College of Engineering,
Andhra University, Visakhapatnam, India

PG Scholar,
[*]Department of Electrical & Electronics Engineering, St. Theresa Institute of Engineering & Technology,
Garividi, Vizianagaram, India

Scientist-B
[$]Naval Science and Technological Laboratory, Defence Research & Development Organization
Visakhapatnam, Andhra Pradesh, India

Professor
[@]Department of Electrical Engineering, Andhra University College of Engineering,
Andhra University, Visakhapatnam, India

Abstract—
These days, the desperate need for energy, provoked the requirement of energy efficient and ergonomic electrical machines suitable for propulsion, wind power applications, etc. This forces a machine designer to their extreme limits in designing their customized machine suitable for their application. Thus, in this quest for energy efficient electrical machines, there came into existence many new design based theories. In this research work the authors sincerely attempt to identify and segregate the Brushless PM motor design parameters from CAD point of view and apply the concept of Interval Arithmetic; so that a valid range(bounds) of the design parameter(s) can be determined and acknowledged before going into a suitable optimization technique for optimizing them. With the help of MATLAB® CLI the bounds for the motor design parameters can be determined in which the optimal design solution lies.

***Keywords*—Brushless PM Motor, Computer Aided Design (CAD), Design Parameters, Interval Arithmetic, Designer Fixed Parameter, Designer Variable Parameters**

I. INTRODUCTION

The design of Brushless PM motor demands new challenging aspects due to their requirement in area like underwater vehicle propulsion [15], wind energy systems, etc., where good ergonomics and energy efficiency come into play. The design involves identification and derivation of the motor design parameters in a procedural manner. In many cases after ending up with all the machine parameters derived, the designer might not have an idea for selecting a range for the parameter(s) to be considered for optimization. At such point, the 'Interval Arithmetic' concept aids the designer to allot a range to these parameter(s) which he wishes to consider for optimization and hence freezing the optimization search space. Interval Arithmetic concept gives an idea of variation of the magnitude of the machine design parameter(s) so that a designer may not go nuts before going for optimization. Interval Arithmetic is an elegant tool for practical work with inequalities, approximate numbers, error bounds, etc. Likewise, if there is uncertainty over a field then interval analysis is the best tool [10],[13],[14], which gives the bound within which an optimal solution may lie and at the same time it gives the maximum and minimum limits of the solution possible of all the dependent design parameters [3].

II. GENERAL SEGREGATION OF BLPM MOTOR DESIGN PARAMETERS

Generally, the parameters that form the basic building block for designing a Brushless PM motor are first derived from their respective magnetic, electric, thermal and drive circuits [11], [12]. There is very less quantum of literature for formulating the equations that govern the design of a BLPM motor [4], [7], [8]. These equations in brief are tabulated as follows in Table I:

TABLE I
GENERAL SEGREGATION OF BLPM MOTOR DESIGN PARAMETERS

S.No.	Parameter	Symbol	Expression
	Electrical		
1	Output Power	P_{out}	
2	Supply Voltage	V	
3	Electrical speed in rad/s	W_e	$(N_m/2)\omega_m$
4	Distribution Factor	K_d	$\sin(N_{spp}\,\theta_{se}/2) / (N_{spp}\sin(\theta_{se}/2))$
5	Coil Pitch	T_c	$\alpha_{cp}\,T_p$
6	Number of Phases	N_{ph}	
	Mechanical		
1	No-load Speed in rps	N_{nl}	
2	Full-load Speed in rps	S_r	
3	Number of Slots	N_s	
4	Number of slots per phase	N_{sp}	N_s / N_{ph}
5	Slots per pole per phase	N_{spp}	
6	Mechanical speed in rad/s	W_m	$2\,\pi\,S_r$
7	Slots per pole	N_{sm}	$N_{spp}\,N_{ph}$
	Magnetic		
1	Permeance Coefficient	PC	
2	Magnetic area	A_m	$\dfrac{\alpha\ \pi\,(R_{ro} - (L_m/2))\,L}{180}$
3	Flux concentration factor	$C_{\varnothing}$	$2\,\alpha_M/(1+\alpha_M)$
4	Air gap area	A_g	$T_pL(1+\alpha_m)/2$
5	Magnetic Leakage factor	K_{ml}	$[1+((4L_m/(\pi\mu_r\alpha_m\,T_p))\ \ln(1+(\pi g/((1-\alpha_m)T_p))))]$
6	Number of Magnet Poles	N_m	
	Geometric		
1	Magnet fraction	α_M	
2	Stator Inner Radius	R_{si}	$R_{ro} + g$
3	Rotor Outer Radius	R_{ro}	
4	Axial Length of motor	L	
5	Radial Air gap length	g	
6	Angular Pole Pitch	θ_p	$2\pi / N_m$
7	Span of magnet in degrees	α	$\alpha_M\,(360 / N_m)$
8	Pole Pitch	T_p	$R_{si}\,\theta_p$
9	Radial length of magnet	L_m	$PC\ g\ C_{\varnothing}$
10	Angular slot pitch	θ_{se}	π / N_{sm}
11	Coil-pole fraction	α_{cp}	

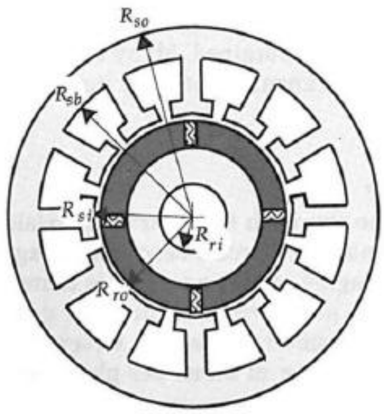

Fig. 1 Radial flux Brushless PM motor topology showing geometrical definitions

III. PROPOSED INTERVAL AND CAD BASED SEGREGATION OF BLPM MOTOR DESIGN PARAMETERS

Any CAD based program of motor design [9], first identifies the inputs given by the user and then the variables to be optimized, upon suitable formulation of an objective function and practical constraints. In this section, the authors have attempted to segregate the machine parameters in a suitable format for CAD based optimization by identifying the motor's 'Variable' and 'Derived' design parameters. Based on the field of application some of these 'Variable' and 'Derived' design parameters can be identified as constraints. The 'Variable' design parameters can be considered as variables in the final objective function for CAD based optimization.

The classification of some of the design parameters shown in Table I does not suit the requirements of a CAD based optimization and interval analysis. Hence, the following classification is proposed:

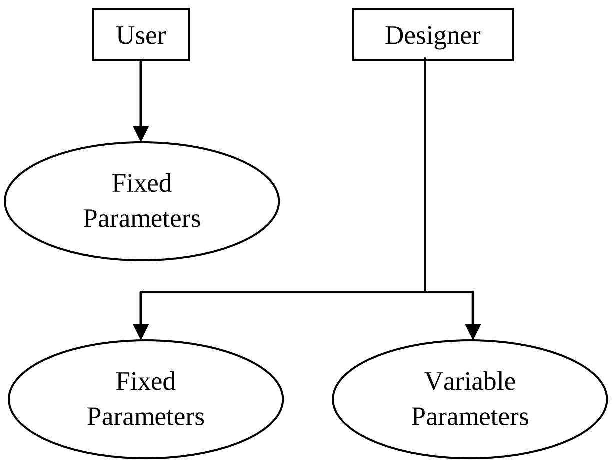

Fig. 2 Tree Diagram of the Proposed Segregation of BLPM Motor design Parameters for Interval and CAD based design optimization

User Fixed Parameters: The "User" (one who wants the design of the motor) fixes some requirements of the motor to suit his purpose like the output power of the motor, supply voltage, etc. These specifications are coined as *"User Fixed Parameters"*.

Designer Fixed Parameters: The Designer (one who takes up the user request for design), after getting the User Fixed Parameters, further divides the other machine design parameters into 'Designer Variable' and 'Designer Derived' parameters. A "*Designer Variable Parameter*" is one, where a designer has the choice to vary (within certain limits) and can use a means for optimization.

Designer Variable Parameters: The "*Designer Derived Parameters*" are those which get "varied" upon the variation of one or more of the "Designer Variable Parameters". These parameters directly reflect final design of the motor.

The proposed classification scheme for the machine design parameters of Brushless PM motor is implemented and the modified table is as follows in Table II:

TABLE II
PROPOSED INTERVAL & CAD BASED SEGREGATION OF BLPM MOTOR DESIGN PARAMETERS

S.No.	Parameter	Symbol	Expression
User Fixed Parameters			
1	Output Power in Watts	P_{out}	-
2	No-load Speed in rps	N_{nl}	-
3	Full-load Speed in rps	S_r	-
4	Supply Voltage	V	-
Designer Variable Parameters			
1	Number of Magnet Poles	N_m	-
2	Number of Phases	N_{ph}	-
3	Radial Air gap length in meters	g	-
4	Permeance Coefficient	PC	-
5	Number of Slots	N_s	-
6	Magnet fraction	α_M	-
7	Rotor Outer Radius in meters	R_{ro}	-
8	Length to Diameter ratio of motor envelop	LD	-
9	Coil-pitch fraction	α_{cp}	-
10	Slot depth fraction	α_{sd}	-
Designer Derived Parameters			
1	Electrical speed in rad/s	W_e	$(N_m/2)\omega_m$
2	Angular Pole Pitch	θ_p	$2\pi/N_m$
3	Number of	N_{spp}	N_{sp}/N_{ph}

	slots per pole per phase		
4	Span of magnet in degrees	α	$\alpha_M\,(360/N_m)$
5	Slots per pole	N_{sm}	$N_{spp}N_{ph}$
6	Distribution Factor	K_d	$\sin(N_{spp}\Theta_{se}/2)$ / $(N_{spp}\sin(\Theta_{se}/2))$
7	Pole Pitch	T_p	$R_{si}\,\theta_p$
8	Magnetic area	A_m	$\frac{\alpha\ \ \pi\,(R_{ro} - (L_m / 2))\,L}{180}$
9	Radial length of magnet	L_m	$PC\ g\ C_Ø$
10	Flux concentration factor	$C_Ø$	$2\,\alpha_M / (1+\alpha_M)$
11	Angular slot pitch	θ_{se}	π/N_{sm}
12	Coil Pitch	T_c	$\alpha_{cp}\,T_p$
13	Air gap area	A_g	$T_pL(1+\alpha_m)/2$
14	Magnetic Leakage factor	K_{ml}	$[1+((4L_m/(\pi\mu_r\alpha_m\ T_p))\ \ln(1+(\pi g/((1-\alpha_m)T_p\ ))))]$
15	Stator Inner Radius	R_{si}	$R_{ro}+g$
16	Number of slots per phase	N_{sp}	N_s/N_{ph}

As an example case, from the above Table II, one of the "Designer variable parameters" is selected from the perspective of CAD optimization and interval arithmetic approach. The Variable is N_m – "number of magnet poles", for which the designer gives a range in an interval. The corresponding Designer Derived Parameters dependent on N_m, listed above are calculated using interval arithmetic and their respective ranges/bounds have been determined and acknowledged.

IV. APPLICATION OF INTERVAL ARITHMETIC TO BLPM MOTOR DESIGN

As considered in the previous section, N_m – the number of poles is given a range by the designer and the affected parameters are also represented in intervals. The rules of interval addition, multiplication, etc. are utilized in calculating and evaluating these machine design parameters and the basic rules are presented in the Appendix. The following gives a tree diagram of the "Designer derived parameters" in Table II showing the dependency on N_m – number of poles.

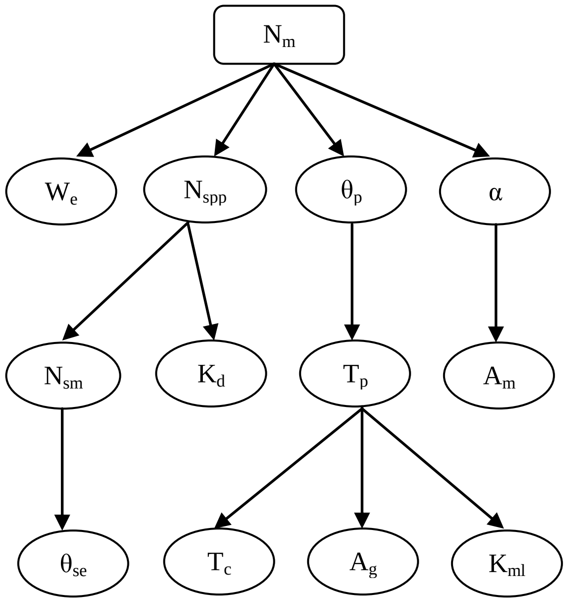

Fig. 3 Tree diagram showing dependency of Designer Variable Parameters on N_m (Designer variable parameter)

V. IMPLEMENTATION OF THE INTERVAL CONCEPT WITH PROPOSED CAD BASED SEGREGATION IN MATLAB®

A program in MATLAB® CLI is written with the help of INTLAB Toolbox, so as to execute and evaluate the various "Designer Derived" parameters both in fixed and interval cases of the 'Designer Variable" parameter, where in the later case utilising the four equations (1), (2), (3) & (4) in Appendix. The results have been verified and presented in a tabular form as follows in Table III:

TABLE IIII
COMPARISON OF THE VALUES OF THE PARAMETERS IN FIXED AND INTERVAL

Parameter	Value in fixed	Value in interval
N_m	36	[32,38]
W_e	4147.3	[3686.4, 4377.7]
θ_p	0.1745	[0.1653, 0.1964]
N_{spp}	1	[0.8771, 1.2501]
A	7.5	[6.6315, 9.0001]
N_{sm}	3	[2.6315, 3.7501]
K_d	1	[0.5112, 1.9026]

T_p	0.0233	[0.0199, 0.0278]
A_m	0.0017	[0.0011, 0.0024]
θ_{se}	1.0472	[0.8377, 1.1939]
T_c	0.0233	[0.0199, 0.0278]
A_g	0.0020	[0.0015, 0.0028]
K_{ml}	1.2956	[1.1279, 1.6489]

This ensures, that for a fixed value of input N_m – number of poles, which lies in the interval for N_m (N_m^l, N_m^u), the corresponding values of the dependent parameters also lies in the interval obtained, ensuring the successful implementation of interval arithmetic to the design parameters of the Brushless PM motor.

The optimal solution set in the final design will lie in these intervals. Upon formulating a suitable objective function and imposing certain meaningful, practical application based constraints and utilizing a suitable optimization procedure, the optimal solution set can be achieved. There are many optimization techniques for such problems, one of them being "Global Optimization Method". This interval optimization technique solved hard mathematical problems arising in the field of qualitative analysis of dynamical systems[2],[5],[6] and discrete geometry, for optimal packing of circles in the square [10],[13].Global optimization methods have also been applied for theoretical chemical problems [1], and for the evaluation of bounding methods [14].

VI. CONCLUSION

The successful segregation approach of Brushless PM motor design parameters from CAD point of view is presented. Using the proposed segregation procedure, a motor designer can straight away identify the various 'Designer Variable' and 'Designer Derived' parameters, where some of these based on the field of application can be identified as constraints and as variables in the objective function. Interval arithmetic can be utilized to determine and have a pre-idea of the bounds of "Designer Derived" parameters of a Brushless PM motor given a bound to the "Designer Variable" parameters. This is very useful for a designer to select appropriately the correct search space in the optimization problem, which is the range of "Designer Variable" parameters going into the optimization problem. If a designer is just able to give the bounds to the input "Designer Variable" parameters the corresponding bounds for all the "Designer Derived" parameters can be determined, giving the flexibility to the designer to cross-check the values of various machine design parameters before going into a suitable optimization scheme. The final optimal solution set always lies in the "interval output" of the corresponding interval parameters. A suitable optimization scheme can then be utilized to optimize the design parameters and hence the final design of the motor. This paper presents the case of only one input design parameter to vary, however it may be applied to a desired number of multiple 'Designer Variable" parameters and correspondingly get bounds for the 'Designer Derived" parameters.

REFERENCES

[1] J. Balogh, T. Csendes, and R.P. Stateva, *Application of a stochastic method to the solution of the phase stability problem: cubic equations of state.* Fluid Phase Equilibria 212 (2003), pp. 257-267.

[2] B. Banhelyi, T. Csendes, and B.M. Garay, *Optimization and the Miranda approach in detecting horseshoe-type chaos by computer.* Int. J. Bifurcation and Chaos 17 (2007), pp. 735-747.

[3] A. Neumaier, *Interval methods for systems of equations*, Encyclopedia of Mathematics and its Applications, Cambridge University Press.

[4] D. Hanselman, *Brushless Permanent Magnet Motor Design*, Magna Physics Publication.

[5] T. Csendes, B. Banhelyi, and L. Hatvani, *Towards a computer-assisted proof for chaos in a forced damped pendulum equation.* J. Computational and Applied Mathematics 199 (2007), pp. 378-383.

[6] T. Csendes, B.M. Garay, and B. Banhelyi, *A verified optimization technique to locate chaotic regions of Henon systems.* J. of Global Optimization 35 (2006), pp. 145-160.

[7] A. Zaki and S. Ibrahim, *"Modeling and Analysis ofPM Brushed DC Motor using FEM"*, EuropeanConference on Power Electronics and Applications,11-14 Sept. 2005.

[8] Z.Q. Zhu, D. Howe and C.C. Chan, "ImprovedAnalytical Model for Predicting the Magnetic FieldDistribution in Brushless Permanent-MagnetMachines", *IEEE Trans. Magnetics*, vol. 38, no. 1,pp. 229-238, Jan 2002.

[9] N. Boules, "Design Optimization of PermanentMagnet DC Motors", *IEEE Trans. IndustryApplications*, vol. 26, no. 4, pp. 786-792,
July/August 1990.

[10] M.Cs. Markot and T. Csendes, *A new verified optimization technique for the "packing circles in a unit square" problems.* SIAM J. on Optimization 16 (2005), pp. 193-219.

[11] K. Hameyer and R.J.M. Belmans, *"Permanent Magnet Excited Brushed DC Motors"*, IEEE Trans. Industrial Electronic, vol. 43, no. 2, pp. 247-255,April 1996.

[12] K. Attalah, Z.Q. Zhu, J.K. Mitchell and D. Howe, *"Curvature Effects in Radial-Field PermanentMagnet Machines"*, Journal of Electric Machinesand Power Systems, vol. 22, pp. 511-520, 1994.

[13] P.G. Szabo et al., *New Approaches to Circle Packing in a Square - With Program Codes.* Springer- Verlag, Berlin, 2007.

[14] B. Toth, J. Fernandez, and T. Csendes, *Empirical convergence speed of inclusion functions for facility location problems.* J. of Computational and Applied Mathematics 199 (2007), pp. 384-389.

[15] Danilo de Carvalho, Teodiano F. Bastos Filho andHansjorg A. Scheenebeli, “Adapting a PMDC BrushMotor to Work Like a Thruster in a ROV”,*Proceedings 43rd IEEE Midwest Symposium onCircuits and Systems*, Lansing MI, Aug 11-18, 2000,Vol. 2, pp.738-742.

APPENDIX

The basic rules one should follow, while using Interval arithmetic are:

If “x” and “y” are two interval numbers, then the rules of addition, subtraction, etc., given in the literature [3] are as follows:

1) Addition in Interval Arithmetic:

$$X + Y = [X_l + Y_l, X_u + Y_u] \quad (1)$$

2) Subtraction in Interval Arithmetic:

$$X – Y = [X_l – Y_u, X_u – Y_l] \quad (2)$$

3) Multiplication in Interval Arithmetic:

$$X*Y = [X_l*Y_l, X_u*Y_u], \quad (3)$$

For all, X and Y > 0

4) Division in Interval Arithmetic:

$$X/Y = [X_l/Y_u, X_u/Y_l], \quad (4)$$

For all, X and Y > 0